THE THEORY OF ELECTRIC CABLES AND NETWORKS

RUSSELL

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By

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PREFACE

THERE is nothing more conducive to the satisfactory working of an electric supply station than having a thoroughly trustworthy and economical network of cables connecting the dynamos with the lamps and motors of the consumer. It is necessary therefore that the engineer have a thorough knowledge of the phenomena connected with the flow of current along conductors and across dielectrics. He must also have a working knowledge of the dielectric strengths of insulating materials and the electric stresses to which they are subjected under working conditions. In addition, the thermal conductivity of the dielectric has to be considered and its effect on the temperature of the conductor.

The author gives some information on these points in this book. His experience in practical testing, and with the difficulties which sometimes arise in interpreting "specifications" and "rules and regulations" has convinced him that the solutions of these problems are of practical use and ought to be more widely known. In fact many of the problems discussed were originally suggested by these difficulties.

Questions in connexion with the electrostatic capacity and the inductance of cables have not been considered, as the author has discussed these points fully in his *Treatise on the Theory of Alternating Currents.* He has also omitted many elementary theoretical considerations as the

reader is supposed to know the elements of the theory of electricity and electrical engineering.

In Chapter I, the fundamental electrical principles are stated and a description is given of the various gauges in use for specifying wires. Conductivity is discussed in Chapter II, and special attention is devoted to the effect of the "lay" on the weight and conductivity of stranded cables. In Chapter III, the standard methods of measuring insulativity are described.

The design of distributing networks is explained in Chapter IV, particular stress being laid on "feeding centres" and on the importance of calculating their positions. The theorems given in this Chapter can easily be expanded so as to enable satisfactory solutions to be obtained for the very complex problems which sometimes arise in practice.

In Chapters V, VI, and VII methods of measuring the insulation resistance of house wiring and distributing networks are given. The author only gives those methods which he has found useful in practice. The problem of the calculation of a suitable resistance to put in the earth connexion with the middle wire was suggested to him by Mr. A. P. Trotter.

The dielectric strength of materials is discussed in Chapter VIII. Unfortunately very few accurate data are yet obtainable, but the author hopes that by applying the methods he suggests, engineers will be able to obtain satisfactory physical "constants" for dielectric strengths. An examination of many published results will show that the experimenters have neglected elementary theoretical considerations which must be taken into account if the results obtained are to be of any value.

In Chapter IX, the grading, and in Chapter X, the heating, of cables is considered. It is only of recent years

that the former of these subjects has been recognized to be of practical importance. In Chapter XI, the very interesting subject of electrical safety valves is considered, but only a few types are discussed, as it is probable that the standard safety device has not yet been evolved.

The author has added a Chapter on lightning conductors, in which he has made extensive use of the classical paper on the subject read to the Institution of Electrical Engineers by Sir Oliver Lodge in 1889.

He has to thank several friends for the kind help they have given him by making suggestions or revising proofs. In particular he has to thank Dr. Chree, F.R.S., for much information about atmospheric electricity and Mr. G. F. C. Searle, F.R.S., for his helpful criticisms of Chapters I and II. He has also to thank Mr. J. N. Alty, A.I.E.E., for his able assistance in drawing the diagrams and reading proofs and his old pupil, the Hon. E. Fulke French, for checking most of the mathematical formulae given.

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CHAPTER I

Fundamental Principles

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In this chapter we shall first give a *résumé* of the elementary electric principles which guide the electrician both in the design of a direct current network for the distribution of electric energy and in the measurements of the electric properties of conducting and insulating materials. We shall also give an account of the various wire gauges used in practice.

Isotropic bodies In discussing the electric properties of conductors it is customary to assume that the conductors are homogeneous in substance, and that the resistance they offer to the flow of current through them is the same in all directions. It has to be remembered, however, that violent mechanical forces like those used in hammering, rolling and wire drawing, produce permanent deformation of the substance of the metal, and alter by varying amounts its electrical properties in different directions. In this chapter we shall assume that the conductors and insulators are isotropic, that is, that their substances

have the same physical properties in all directions. Carefully annealed copper may be considered to be practically isotropic as no tests we can apply can detect any difference in its physical properties in different directions. Carefully annealed glass also is practically isotropic. As all physicists, however, now accept the theory of the molecular structure of bodies, we must admit that if the portion examined were so small that it contained only a few molecules of the substance, the tests for isotropy would not be satisfied. All substances are in fact irregular when the dimensions of the portion examined are comparable with the dimensions of a molecule of the substance.

Ohm's law

If R be the resistance of an electric circuit, E the electromotive force round it, and C the current flowing in it, Ohm's law states that—

$$C = E/R \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1),$$

where the current is measured in amperes, the electromotive force in volts and the resistance in ohms.

In general, if r be the resistance of a part of a circuit containing sources producing a resultant electromotive force E, and if V_1 and V_2 be the potentials of A and B the ends of this portion of the circuit, we have

$$Cr = V_1 - V_2 + E \quad \ldots \quad \ldots \quad \ldots \quad (2).$$

In this equation the current C is positive when it flows from A to B, and E is positive when it tends to produce a current in the same direction.

Equation (2) shows that it is possible to have the potentials V_1 and V_2 at two points on one member of a network of conductors the same, and yet have a current E/r flowing from one point to the other. In this case the local electromotive force E is entirely expended in maintaining the current C flowing through the resistance r. It is obvious that we may short-circuit the two

points without affecting in the least the working of any part of the network. We can also have a current flowing from a point of lower potential through a source of electromotive force to a point at a higher potential.

Joule's law By the definition of electromotive force given in treatises on electricity it follows that if a current of C amperes flow for t seconds through a wire having a potential difference of V volts between its terminals, the work done will be VCt joules. If all this work be expended in heating the wire, that is, if no mechanical work, such as causing an armature to rotate, and no chemical work, such as charging accumulators, is done, we have

$$JH = VCt \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3),$$

where H is the number of water gramme Centigrade units of heat (calories) developed, and J is the number of joules in a calorie. The law expressed by this equation is called Joule's law, as he was the first to employ it to determine the mechanical equivalent of heat. The value of J is very approximately 4·18 ergs per water gramme degree Centigrade, and hence, by Ohm's law, we can write

$$JH = C^2rt = (V^2/r)t \quad \ldots \quad \ldots \quad \ldots \quad (4),$$

or

$$H = 0{\cdot}239\ C^2rt = 0{\cdot}239(V^2/r)t \quad \ldots \quad \ldots \quad (5),$$

approximately.

Since the work done in t seconds is VCt joules, when V and C are maintained constant, it follows that the rate at which work is being done is VC joules per second, that is VC watts.

Example As an illustration of the application of (5), we shall find the rise of temperature per second in a coil formed of a copper wire 0·1 mm. in radius and 1,000 metres long when placed between the hundred volt mains, supposing that no heat is lost by radiation and neglecting the effect of the rise of resistance due to rise of temperature.

At 16° C., the resistance of this wire would be about 508 ohms. By (5), the heat which is generated per second is

$$0{\cdot}239\,(100)^2\,/508,$$

that is, 4·7 calories nearly. Assuming the specific gravity of copper to be 8·9, the mass of the copper wire will be $8{\cdot}9 \times \pi(0{\cdot}01)^2 \times 100{,}000$, that is, 280 grammes nearly. Hence taking the specific heat of copper to be 0·095 the rise of temperature per second will, on the given assumptions, be $4{\cdot}7/(280 \times 0{\cdot}095)$, that is, 0·18° C. nearly.

We conclude, therefore, that, when the coil is connected with the mains, its temperature rises initially by about 0·18° C. per second. As it warms, the heat lost by radiation gradually increases and so the rate at which the temperature rises gradually diminishes until the temperature attains a steady value, when the rate at which heat is being lost by radiation equals the rate at which heat is being generated in the wire.

Resistances in series

If we have n coils of resistance $r_1, r_2, \ldots r_n$ respectively, connected in series, and if C be the current flowing through them, we have, by (2),

$$Cr_1 = V_1 - V_2,\ Cr_2 = V_2 - V_3,\ \ \ldots\ \ Cr_n = V_n - V_{n+1} \quad \ldots \quad (6),$$

where V_p and V_{p+1} are the potentials at the ends of the resistance r_p. It follows, by adding equations (6), that

$$C(r_1 + r_2 + \quad \ldots \quad \ldots \quad + r_n) = V_1 - V_{n+1}.$$

We see, therefore, that $r_1 + r_2 + \quad \ldots \quad + r_n$ is the resultant resistance R of the n coils, so that

$$R = r_1 + r_2 + \quad \ldots \quad + r_n \quad \ldots \quad \ldots \quad \ldots \quad (7).$$

Hence the resistance of n coils, in series equals the sum of the resistances.

Kirchhoff's first law

When we have n conductors connected with a point O and the currents in them have attained their steady values, the algebraical sum of all the currents in these conductors must be zero. For if not,

the quantity of electricity at O would continually increase or continually diminish, which is obviously impossible. In algebraical symbols we may express Kirchhoff's first law as follows—

$$C_1+C_2+\ \ldots\ +C_n=0 \quad \ldots \quad \ldots \quad \ldots \quad (8),$$

or simply, $\Sigma C=0,$

a current C_p being positive when it is flowing towards the common junction.

The potential of the common junction Let V be the potential of the common junction of n arms of a network, and let V_1, V_2, .. V_n, be the potentials of the other ends of the arms, then, by (2) and (8), we have

$$(V_1-V+E_1)/r_1+(V_2-V+E_2)/r_2+\ \ldots\ +(V_n-V+E_n)/r_n$$
$$=0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9),$$

and thus, $V\Sigma(1/r)=\Sigma(V_p+E_p)/r_p.$

In analogy with the nomenclature of alternating current theory, conductors having a common junction will be said to be star connected.

Minimum heating If we assume that the potentials at the ends of n branches, star connected, of a network are constant, and that their resistances and the electromotive forces in their circuits are also constant, we see almost at once, by the differential calculus, that (9) determines the value of V which makes

$$\Sigma(V_p-V+E_p)^2/r_p \text{ or } \Sigma C_p{}^2 r_p$$

a minimum. Hence the actual value of the potential of the common junction is the theoretical value which makes the heating as determined by Joule's law a minimum.

Conductors in parallel When we have n resistances r_1, r_2, .. r_n, connected in parallel between two mains each of negligible resistance, and when the potential difference V between the mains is constant, then the cur-

rents C_1, C_2, .. C_n, in the resistances are given by

$$C_1 = V/r_1,\ C_2 = V/r_2,\ \ldots\ \ldots\ ,$$

and therefore,

$$V = C_1 r_1 = C_2 r_2 = \ \ldots\ = C_n r_n \quad \ldots \quad \ldots \quad (10).$$

Now if C be the current in the main, we have, by Kirchhoff's first law,

$$C = C_1 + C_2 + \ \ldots\ + C_n,$$

and hence, by (10),

$$C = V\,(1/r_1 + 1/r_2 + \ \ldots\ + 1/r_n) \quad \ldots \quad (11).$$

If R be the value of the single resistance which when placed between the mains would allow a current C to flow, we have

$$C = V/R,$$

and thus, by (11),

$$1/R = 1/r_1 + 1/r_2 + \ \ldots\ + 1/r_n = \Sigma 1/r_p \quad \ldots \quad (12).$$

Hence the reciprocal of R equals the sum of the reciprocals of the resistances of the coils.

We shall call R the equivalent resistance.

Since $V = CR$, we find by (10),

$$C_1 = C\,(R/r_1),\ C_2 = C(R/r_2) \quad \ldots \quad \ldots \quad (13).$$

Kirchhoff's second law

If in a network we take any of the conductors which form a closed circuit, the algebraical sum of the currents multiplied by the resistances of these conductors equals the algebraical sum of the electromotive forces round the closed circuit. This theorem was enunciated by Kirchhoff and is known as his second law. It follows at once from (2), for

$$\Sigma Cr = \Sigma\,(V_p - V_{p+1} + E_p) = \Sigma E \quad \ldots \quad \ldots \quad (14),$$

since in a closed circuit $\Sigma(V_p - V_{p+1}) = V_1 - V_2 + V_2 - V_3 + \ \ldots\ + V_n - V_1 = 0$.

Minimum heating of a loop of a network

If in a network we choose any system of conductors which form a closed circuit and if the resultant electromotive force round this circuit be zero, then, for all values of the currents in

these conductors which are consistent with Kirchhoff's first law, the values which make the heating of the conductors a minimum satisfy Kirchhoff's second law.

Let r_1, r_2, .. be the resistances of the various conductors and let C_1, C_2, .. be the currents in them. Since $C_1 - C_2$ gives the resultant value of the currents flowing into or out of the circuit at the common junction of r_1 and r_2, and as this value is to be the same whatever hypothetical values we give to the currents, we see that these values must be C_1+x, C_2+x, C_3+x, .. where x may be positive or negative. The total heating W, therefore, is given by

$$W=(C_1+x)^2r_1+(C_2+x)^2r_2+ \;..\; +(C_n+x)^2r_n$$
$$=\Sigma C^2r+2x\Sigma Cr+x^2\Sigma r.$$

But by Kirchhoff's second law $\Sigma Cr = \Sigma E$ and is therefore zero. Hence

$$W=\Sigma C^2r+x^2 \Sigma r,$$

and W, therefore, has its minimum value ΣC^2r when x is zero, that is, when the values of the currents are in accordance with Kirchhoff's second law.

Volume resistivity The volume resistivity ρ of a conducting substance at a given temperature is the resistance offered at that temperature by a centimetre cube of the substance to a flow of electricity from one face to the opposite face of the cube, the lines of flow being perpendicular to these faces. In practice, ρ is usually expressed in microhms (millionths of an ohm).

Since, by Ohm's law, the fall of potential from one face to the other of the cube is uniform it follows that the resistance of a rectangular prism one square centimetre in cross section and the nth part of a centimetre long is ρ/n. We see that the resistance of a rectangular prism, one square cm. in cross section and l cms. long is the same

as that of nl prisms of the same section and of length $1/n$ arranged in series. It is therefore $nl \times (\rho/n)$, that is, ρl. If we now suppose this prism divided up into m parallel prisms, the areas of the ends of which will be the mth part of a square centimetre, the resistance of each of these elementary prisms will be $m\rho l$. If we have a prism of length l and cross sectional area S, we may suppose it to consist of mS elementary prisms arranged in parallel. Its resistance would therefore be $m\rho l/(mS)$, that is $\rho l/S$. If R, therefore, be the resistance of this prism in microhms we have

$$R = \rho l/S \quad \ldots \quad (15).$$

It has to be remembered that this is true also for cylindrical conductors of any section since a cylinder is a particular case of a prism. The only assumption made is that the current flow is parallel to the axis.

Section variable When the section of a wire varies slightly, it is customary in calculating its resistance to measure the cross sectional areas at *equidistant* points along the wire, and to substitute the mean of the values thus found for S in formula (15). To see the nature of the error made by this assumption, let us consider the resistance R of a series of n cylinders each of length l/n and of cross sectional areas $S_1, S_2, \ldots S_n$. We shall suppose that the cylinders are joined to one another by a material infinitely thin, having absolutely no resistivity, and spread uniformly over their ends. This will ensure that the flow of the current at every point in each of the cylinders is parallel to its axis and hence we can at once write down the value of all the resistances. The actual value of the resistance of the whole will be greater than this, for the stream lines of current will be curved. We have, by (15),

$$R = (\rho l/n)\{1/S_1 + 1/S_2 + \ldots + 1/S_n\}.$$

The formula ordinarily used is

$$R' = \rho l / \{(S_1+S_2+ \quad .. \quad +S_n)/n\}$$
$$= \rho l n/(S_1+S_2+ \quad .. \quad +S_n).$$

Now, by algebra, since $S_1+S_2+ \quad .. \quad +S_n$ is greater than $n(S_1 S_2 \quad .. \quad S_n)^{1/n}$ and, for the same reason, $1/S_1+1/S_2+ \quad .. \quad +1/S_n$ is greater than $n(S_1 S_2 \quad .. \quad S_n)^{-1/n}$, we have, therefore, $(S_1+S_2+ \quad .. \quad +S_n)(1/S_1+1/S_2+ \quad .. \quad +1/S_n)$ greater than n^2, and hence R is greater than R', provided that S_1, S_2, .. are not all equal. Since the actual value of the resistance of the n rods in series is greater than R, it is *à fortiori* greater than R'. Therefore the value of the resistance of a wire calculated by means of (15) by making the customary assumptions is too small. Conversely the value of the volume resistivity, calculated from the value of the resistance found by a Wheatstone's bridge by aid of (15) is too great. If the wire be nearly uniform in cross section the error due to neglecting the curvature of the lines of flow is very small.

Volume resistivities of metals

In the following table the values of the volume resistivities of pure metals at 0° C., found by J. Dewar and J. A. Fleming (*Phil. Mag.* p. 299, Sept. 1893) are given. The metals were in all cases soft and annealed.

Metal	ρ_0 microhms	Metal	ρ_0 microhms
Aluminium . . .	2·665	Nickel	12·32
Cadmium	10·02	Palladium . . .	10·22
Copper	1·561	Platinum . . .	10·92
Gold	2·197	Silver	1·468
Iron	9·065	Thallium . . .	17·63
Lead	20·38	Tin	13·05
Magnesium . . .	4·355	Zinc	5·751

Conductance and conductivity

The conductance K of a conductor is measured by the current flowing in the conductor when unit potential difference is applied at its terminals. Hence, by Ohm's law,

$$C=V/R=VK,$$

and so, $K=1/R$.

The conductivity κ of the substance of a conductor is measured by the current which flows, parallel to an edge, through a unit cube of the substance, when unit difference of potential is maintained between the two faces perpendicular to the edge. Hence it readily follows that the conductance K of a wire of conductivity κ, length l, and cross section S, is given by

$$K=\kappa(S/l).$$

As K and κ are simply the reciprocals of R and ρ, it is unnecessary to tabulate their values as the values of the latter quantities for various wires and substances are given in tables. It is also unnecessary to discuss methods of measuring conductivity separately from methods of measuring resistivity, as any method which measures the one quantity will also give the other.

Circular mil

We shall now describe how the dimensions of the conductors used in practice are specified. On the Continent of Europe, thin wires are usually specified in terms of their diameters measured in millimetres. In England and America they are generally specified in terms of certain gauges or in terms of the diameters measured in mils, a mil being the thousandth part of an inch. Cable manufacturers call the area of a circle one mil in diameter a "circular mil." If, for instance, the diameter of a wire were d mils, its area would be d^2 circular mils or $0{\cdot}7854\ d^2/(1{,}000)^2$ square inches approximately, since the value of a circular mil is $0{\cdot}7854/(1{,}000)^2$ square inches. In practice,

it is convenient to use the expression circular mil as it is a perfectly definite unit and by its use we avoid the necessity of multiplying the square of the diameter by $\pi/4$, i.e. by 0·7854.

Gauges Various gauges are used for the measurement of wires. The oldest of them is the Birmingham Wire Gauge (B.W.G.). In this gauge the thickest wire which is tabulated has a diameter of 500 mils and is denoted by No. 00000. The thinnest wire has a diameter of 4 mils and is called No. 36. In England this gauge has been replaced by the British Legal Standard or as it is generally called the Standard Wire Gauge (S.W.G.). As in the B.W.G., the thickest wire which is tabulated has a diameter of 500 mils, but it is denoted by No. 0000000 or 7/0. The thinnest wire is No. 50 and is 1 mil in diameter. From the tables given below it will be seen that the diameters of the wires corresponding to the various numbers do not proceed by any regular law. The number of sizes is ample for all practical purposes. In electric lighting practice, conductors having a larger sectional area than that of a No. 14 S.W.G. wire are stranded. The trolley wires used in electric traction are generally No. 0, 3/0, or 4/0, S.W.G. and have diameters of 324, 372, and 400 mils respectively.

In America, the Brown and Sharpe Gauge (B. & S.) is in general use. It is the only gauge that has been calculated on scientific principles. It is founded on the Birmingham Wire Gauge but the diameters, and consequently also the areas, of the cross sections of the wires corresponding to the various numbers are in geometrical progression. The largest wire is 4/0 and has a diameter of 460 mils. The smallest is No. 40 with a diameter of 3·14 mils. The diameter of every wire in this gauge is practically double

that of the sixth consecutive wire succeeding it or half that of the sixth consecutive wire preceding it. It follows that the area of the cross section of every wire is practically half that of the area of the wire which is three above it, or double that of the wire which is three below it. For instance :—

B. & S. Gauge	Diameter in mils	Area in Circular mils	Mass in lbs. of 1,000 yds. Cu. wire
No. 0 . . .	325	105,600	958
No. 3	229	52,630	477
No. 6	162	26,250	238

It will also be seen that the weight of a yard of No. n wire will be half that of a yard of No. $(n—3)$ wire and double that of No. $(n+3)$ wire.

To find the value of the ratio x of the diameters of consecutive wires in this gauge, let us calculate from the diameters of No. 0 and No. 10 wire respectively. These are 324·95 and 101·89 mils. We have, therefore,

$$324{\cdot}95 = 101{\cdot}89\, x^{10}$$

and hence, $10 \log x = \log 324{\cdot}95 — \log 101{\cdot}89 = 0{\cdot}5036849$

and thus, $x = 1{\cdot}123$ very nearly.

The diameter of 4/0 wire would be given by $324{\cdot}95\,(1{\cdot}123)^3$, that is, 460·2 mils nearly. The diameter of No. 40 wire would be $324{\cdot}95\,(1{\cdot}123)^{-40}$ which equals 3·14 mils nearly.

The following table gives the diameters of the wires in the Standard Wire Gauge, the Birmingham Wire Gauge, and Brown and Sharpe's Gauge. The masses of a thousand yards of a pure copper wire of the various sizes are given in the second table for purposes of comparison.

THE ENGLISH AND AMERICAN GAUGES (IN MILS).

No.	S.W.G.	B.W.G.	B. & S.	No.	S.W.G.	B.W.G.	B. & S.
4/0	400	454	460·2	19	40	42	35·9
3/0	372	425	409·6	20	36	35	32·0
2/0	348	380	364·8	21	32	32	28·5
0	324	340	324·9	22	28	28	25·3
1	300	300	289·3	23	24	25	22·6
2	276	284	257·6	24	22	22	20·1
3	252	259	229·4	25	20	20	17·9
4	232	238	204·3	26	18	18	15·9
5	212	220	181·9	27	16·4	16	14·2
6	192	203	162·0	28	14·8	14	12·6
7	176	180	144·3	29	13·6	13	11·3
8	160	165	128·5	30	12·4	12	10·0
9	144	148	114·4	31	11·6	10	8·9
10	128	134	101·9	32	10·8	9	7·9
11	116	120	90·7	33	10·0	8	7·1
12	104	109	80·8	34	9·2	7	6·3
13	92	95	72·0	35	8·4	5	5·6
14	80	83	64·1	36	7·6	4	5·0
15	72	72	57·1	37	6·8	—	—
16	64	65	50·8	38	6·0	—	—
17	56	58	45·3	39	5·2	—	—
18	48	49	40·3	40	4·8		

MASS OF 1,000 YARDS OF COPPER WIRE IN POUNDS, WHEN ITS SPECIFIC GRAVITY IS 8·90.

S.W.G. No.	Mass Lbs.	S.W.G. No.	Mass Lbs.	S.W.G. No.	Mass Lbs.	S.W.G. No.	Mass Lbs.
4/0	1452	8	232·3	19	14·52	30	1·395
3/0	1256	9	188·2	20	11·76	31	1·221
2/0	1099	10	148·7	21	9·293	32	1·058
0	952·7	11	122·1	22	7·115	33	0·9076
1	816·8	12	98·16	23	5·228	34	0·7682
2	691·3	13	76·82	24	4·393	35	0·6404
3	576·3	14	58·08	25	3·630	36	0·5242
4	488·5	15	47·05	26	2·940	37	0·4196
5	407·9	16	37·17	27	2·440	38	0·3267
6	334·6	17	28·46	28	1·987	39	0·2454
7	281·1	18	20·91	29	1·679	40	0·2091

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CONDUCTIVITY

CHAPTER II

Conductivity

The elastic constants of metal wires—Hard drawn and annealed copper—The density of copper—The standard density—Mass resistivity—Resistance temperature formulae—Resistivity temperature formulae—Numerical example—Tinning—Measuring the rise of temperature—Temperature coefficients of metals—Stranded cables—Effect of lay on the mass of the conductor—Effect of lay on the resistance of the conductor—Permissible current in cables—Resistance of cables—High frequency alternating currents—Data for calculations—References.

The elastic constants of metal wires

THE elasticity of an isotropic metal depends on two qualities of the metal, its resistance to change of volume and its resistance to change of shape. The former depends on the compressibility and the latter is called the rigidity. If a piece of metal recovers its original volume and shape exactly when the forces applied to it are removed, it is said to have been strained within the limits of perfect elasticity. Within these limits Hooke's law—that the effects produced are proportional to the applied forces—is true, and we may speak of the metal as being perfectly elastic. It is of importance that engineers should know the elastic constants of metals as these are an indication of their suitability for certain purposes.

If when a body is subjected to any forces every cubical portion of it remains a cube, although its volume has altered, this strain is called a compression when the volume has diminished, and an expansion when the

volume has increased. The bulk modulus k of the substance is the ratio of the stress to the strain. If a stress of dp dynes per square centimetre uniformly applied to the surface of a body of volume V alter its volume to $V-dV$, the compression is measured by dV/V, and thus, by Hooke's law

$$k = \text{stress/strain} = -dp/(dV/V) = -V(dp/dV).$$

In the preceding case we have considered change of volume but not change of shape. We shall now consider change of shape without change of volume. If two pairs of the bounding faces of every elementary cubical portion of a piece of metal remain squares while the other pair of faces become parallelograms having angles $90°+A$ and $90°-A$ respectively, the piece of metal is said to be sheared. A simple way of producing a shear on a cube is to apply tangential stresses of p dynes per square centimetre to the four faces which remain squares, the stresses on opposite faces being oppositely directed. If θ be the circular measure of A, θ will equal $\pi A/180$, and the rigidity n is given by

$$n = \text{stress/strain} = p/\theta \text{ dynes per square centimetre.}$$

As an example, let us suppose that a tangential stress of 10^8 dynes per square centimetre produces a shear of $\pi \times 10^{-4}$ radians in copper. In this case

$$n = (1/\pi) \times 10^{12} = 3{\cdot}18 \times 10^{11} \text{ dynes cm.}^{-2} \text{ approx.}$$

If a uniform stress of T dynes per square centimetre of cross section pull out a uniform rod from a length l to a length $l+\lambda$ cm., we have ;—

$$\text{Young's modulus} = E = \text{stress/strain} = T/(\lambda/l) \text{ dynes cm.}^{-2}.$$

These constants are not independent of one another. It is proved in treatises on elasticity that for an isotropic material

$$1/E = 1/(9k) + 1/(3n),$$

and therefore E cannot be greater than $3n$.

G. F. C. Searle (*Phil. Mag.* p. 199, Feb. 1900, or *Experimental Elasticity*, p. 113) gives the following values of E and n for various metals and alloys, obtained on the assumption that the wires experimented on were isotropic :—

Metal	E dynes cm.$^{-2}$	n dynes cm.$^{-2}$
"Silver" Steel	$1{\cdot}98 \times 10^{12}$	$7{\cdot}87 \times 10^{11}$
Brass (hard drawn)	1·02	3·72
Phosphor Bronze	1·20	4·36
Silver (hard drawn)	0·78	2·82
Copper (hardened by stretching)	1·24	3·88
Copper (annealed)	1·29	4·02

In the last two cases E is slightly greater than $3n$. This shows that the wires were not strictly isotropic.

As a rule there is an alteration of temperature when the volume or shape of a body is altered. Hence, strictly speaking, the values of the elastic constants are indeterminate unless the alteration, if any, of the temperature is specified. It is customary to consider two cases, namely, (1) when no heat is allowed to enter or leave the body during the application of the forces, and (2) when the temperature of the solid is maintained constant. The constants obtained in the first case are the adiabatic constants k_ϕ, E_ϕ and n_ϕ, and in the second case the isothermal constants k_t, E_t and n_t. It may be shown by aid of the principles of thermodynamics that in the case of copper at 0° C., k_ϕ is about 3 per cent. greater than k_t, that E_ϕ is about 0·3 per cent. greater than E_t, and that n_ϕ and n_t have practically the same value. For most purposes, therefore, we may disregard the differences between E_t, n_t and E_ϕ, n_ϕ.

Hard drawn and annealed copper

In making copper wire, a drawplate of hard steel pierced with several holes of graduated sizes is mounted on the draw-bench. The wire

is drawn in succession through smaller and smaller holes which are widest where the wire enters and taper slightly to where it leaves. During each operation it is wound on a reel on the draw-bench. After this process the wire is *hard-drawn.* It is usually circular in section having been drawn through conical holes. By properly designing, however, the holes in the drawplate, the wire can be drawn so as to have a cross section of any desired shape.

Copper is annealed by heating it to redness and cooling it suddenly. The result is that it is rendered soft and malleable. In electrical work it is customary to divide copper wires into "hard drawn" and "annealed." The Engineering Standards Committee (England) define a hard drawn copper wire as one which will not elongate by more than 1 per cent. without fracture. Such wire is generally used for overhead conductors where mechanical strength is desirable, as its breaking stress is about double that of annealed copper, and its conductivity is only about 2 per cent. less than that of the soft copper wires used for insulated conductors in electric lighting.

Benton's experiments

J. R. Benton has made an investigation (*Physical Review*, vol. xiii, p. 234, 1901) on the effect of "drawing" on the elasticity of copper wire. The copper wire was first annealed by heating it electrically to redness and cooling it suddenly. The effect of successive drawings on its elastic constants, determined on the assumption that the "drawn" wire was isotropic, was then investigated. The wire was finally annealed and its constants were again found.

Treatment	Wire	Diameter in cm.	E dynes cm.$^{-2}$	n dynes cm.$^{-2}$
Annealed	A			
Drawn once.		0·1504		$4{\cdot}017\times10^{11}$
,, twice.		0·1391		3·946
,, 3 times		0·1306	$1{\cdot}387\times10^{12}$	3·919
,, 6 ,,		0·1122	1·390	3·876
,, 9 ,,		0·0941	1·420	3·863
Re-annealed		0·0932	1·190	4·322
Annealed	B	0·1617	1·282	4·177
Drawn once		0·1508	1·321	4·015
,, 3 times		0·1366	1·290	3·961
,, 5 ,,		0·1230	1·326	3·948
,, 7 ,,		0·1140	1·340	3·945
,, 9 ,,		0·1001	1·332	3·897
Re-annealed		0·0986	1·109	4·305

These results apparently indicate that the effect of successive drawings is to increase the value of Young's modulus and to diminish the value of the rigidity, but since E is in most cases greater than $3n$ the assumption of isotropy is not strictly permissible. On re-annealing the wire the value of Young's modulus is appreciably smaller than its initial value, and the value of the rigidity is appreciably greater.

The density of copper

Instead of considering how the electrical resistance of a copper wire varies with the length and the area of the cross section, it is often convenient in practice to consider how it varies with the length and the mass of the wire. The experiments of Fitzpatrick (*B. A. Report*, 1894) prove that the densities (mass in grammes per cubic centimetre) of most kinds of commercial copper at ordinary temperatures lie between 8·90 and 8·95.

Since the coefficient of linear expansion of copper for rise of temperature is 0·0000168, we have

$$l_t = l_o\{1+0{\cdot}0000168t\},\text{ approximately,}$$

where l_o, l_t are the lengths of a copper wire at 0° and t° C. The volume V_t of a lump of copper at t° is, therefore, given by

$$V_t = V_o\{1+0{\cdot}0000168t\}^3$$
$$= V_o\{1+3\times 0{\cdot}0000168t\} \text{ approximately}$$
$$= V_o\{1+0{\cdot}0000504t\}.$$

Hence, the volume of a given mass of copper increases by about the half of 1 per cent. for a rise in temperature of 100° C. Thus the density of copper varies appreciably with the temperature, diminishing by about 0·005 per cent. per degree as the temperature rises.

The standard density

In order to simplify the arithmetical work necessary in making calculations and to assist the memory, it is customary in England to assume that copper weighs 555 lb. per cubic foot at 60° F. (15·6° C.). Hence the volume of 555 lb. of copper at 0° C. is taken to be 1,728/(1+0·0000504 × 15·6), that is, 1,726·6 cubic inches. Hence at 0° C. 1 lb. of copper has a volume of 3·111 cubic inches and 1 cubic inch has a mass of 0·3214 lb. at the same temperature.

Since there are 453·6 grammes in a pound and 16·39 cubic centimetres in a cubic inch, it will be seen that the standard density of copper at 15·6° C. is taken as 555 × 453·6/(1,728 × 16·39), that is, 8·890. The standard density of copper at 0° C. is, therefore, taken as 8·89 (1+0·0000504 × 15·6), that is, 8·897 approximately.

Mass resistivity

The resistance in ohms at 60° F. of a wire one metre long and weighing one gramme is called the mass resistivity of the metal forming the wire. We shall denote mass resistivity by ρ'. For annealed high conductivity copper the standard value of ρ' assumed in England is 0·1508, and for hard-drawn high conductivity copper it is taken to be 0·1539.

If the mass of a wire be m grammes and its length be L metres, then, if ρ' be the mass resistivity and R the resistance of the wire, we have

$$R = \rho'(L^2/m).$$

To prove this, notice that the resistance of a metre of the wire weighing m/L grammes would be $\rho'/(m/L)$, i.e. $\rho' L/m$, and hence the resistance of L metres of this wire, having a total weight of m grammes would be $\rho' L^2/m$ ohms.

We have already seen that if ρ be the volume resistivity of the copper (p. 10),

$$10^6 R = \rho L 10^2/S,$$

where R is measured in ohms, ρ in microhms, L in metres, and S in square centimetres.

Hence, if V be the volume of the copper and d its density (8·89) at 60° F.,

$$10^6 R = \rho L^2 10^4/V$$
$$= \rho L^2 d 10^4/m.$$

Therefore, $\quad 10^6 \rho'(L^2/m) = \rho\,(L^2/m)\ 8{\cdot}89 \times 10^4,$

and hence, $\quad \rho' = \rho \times 0{\cdot}0889,$

and $\quad \rho = \rho' \times 11{\cdot}25$, at 60° F.

For annealed high conductivity copper, for instance,

$$\rho = 0{\cdot}1508 \times 11{\cdot}25$$
$$= 1{\cdot}6965, \text{ at } 60^\circ \text{ F.},$$

and for hard drawn high conductivity copper,

$$\rho = 0{\cdot}1539 \times 11{\cdot}25$$
$$= 1{\cdot}731, \text{ at } 60^\circ \text{ F.}$$

These values of ρ and ρ' are taken as the standard values in England, and are generally referred to as Matthiessen's Standards. It has to be remembered, however, that they are only consistent with one another when the specific gravity of the copper is 8·90. But the values of the specific gravities met with in practice may vary

from 8·88 to 8·96. It will be seen, therefore, that the percentage conductivity of a sample of copper wire in terms of Matthiessen's Standard as ordinarily determined may vary by as much as the half of 1 per cent. according as we take the mass resistivity or the volume resistivity standard. The conductivity of the best quality copper used in practice is often 2 or 3 per cent. greater than "Matthiessen's Standard."

Resistance temperature formulae

The resistance of copper wire alters considerably with temperature. As the result of an extensive series of experiments Matthiessen (*Phil. Trans., Roy. Soc.*, 1862) gave the following formula for the connexion between the conductance K_t of a wire at $t°$ C. and its conductance K_o at 0° C.—

$$K_t = K_o[1 - 0{\cdot}0038701t + 0{\cdot}000009009t^2].$$

Hence, by means of the binomial theorem, it is easy to show that the formula connecting the corresponding resistances R_t and R_o is,

$$R_t = R_o[1 + 0{\cdot}00387t + 0{\cdot}0000060t^2 - 0{\cdot}000000012t^3 - \ldots].$$

As the error introduced by the assumption that the relation between R_t and R_o is a linear one, at least up to 50° C., is not large, this assumption is generally made in practice. We assume, therefore, that

$$R_t = R_o(1 + at)$$

and give such a value to a that the errors due to this assumption are as small as possible. The following values of a, found experimentally, were quoted by Dr. Glazebrook in the *Electrician*, vol. 59, p. 65.

Observers	Range in Deg. Cent.	a
Matthiessen in *Phil. Trans.* 1862 . .	0–50 0–100	0·00412 0·00422
(These mean values of a are deduced from Matthiessen's formula.)		
Dewar and Fleming, *Phil. Mag.* 1893.	0–18 0–60 0–100 0–200	$0{\cdot}0042_4$ $0{\cdot}0042_7$ $0{\cdot}0042_8$ $0{\cdot}0042_6$
Swan & Rhodin, *Proc. Roy. Soc.* 1894	13–90	$0{\cdot}0040_8$ $0{\cdot}0041_6$
Fitzpatrick, *B. A. Report*, 1894. .	—	$0{\cdot}0040_5$

For temperatures between 0° and 50° C. the value customarily taken for a in England in 1908 is 0·00428. In America and Germany 0·0042 is taken as the standard value. It is probable that a varies appreciably with the kind of copper used and the method of treatment to which it has been subjected, but sufficient data on this point have not yet been obtained. We shall take 0·0042 as the standard value for the temperature coefficient—for temperatures from 0° to 50° C. The formula is therefore—

$$R_t = R_o(1+0{\cdot}0042t).$$

Resistivity temperature formulae

Assuming that the formula

$$R_t/R_o = 1 + at$$

gives the value of R_t accurately, we shall find the temperature coefficients for the volume resistivity and for the mass resistivity. For the volume resistivity, we have

$$\rho_t = R_t(S_t/l_t) \text{ and } \rho_o = R_o(S_o/l_o),$$

and thus, $\rho_t/\rho_o = (R_t/R_o)(S_t/S_o)(l_o/l_t)$

$$= (1+at)(1+\gamma t)^2/(1+\gamma t),$$

where γ is the coefficient of the linear expansion of copper for rise of temperature. Hence

$$\rho_t=\rho_o\{1+(a+\gamma)t\} \text{ approximately.}$$

We also have $\rho'_t=R_t(m/L^2_t)$ and $\rho'_o=R_o(m/L^2_o)$,

and thus,
$$\rho'_t=\rho'_o(R_t/R_o)/(L^2_t/L^2_o)$$
$$=\rho'_o(1+at)/(1+\gamma t)^2$$
$$=\rho'_o\{1+(a-2\gamma)t\} \text{ approximately.}$$

If $a=0{\cdot}00420$ and $\gamma=0{\cdot}000017$, we may write

$$\rho_t=\rho_o(1+0{\cdot}00422t) \text{ and } \rho'_t=\rho'_o(1+0{\cdot}00416t).$$

It is customary to assume that the values of the temperature coefficients for R_t, ρ_t and ρ'_t are the same. We see that the maximum error which arises from this neglect of the thermal expansion of the metal, when t is less than 50° C., is less than 0·2 per cent.

Numerical example

As an application of the formulae we shall consider the problem of finding the percentage conductivity in terms of "standard" copper of a copper rod one centimetre in diameter. We shall suppose that when a current is flowing through it, the potential difference between two knife edges at a distance of 200·0 cms. apart as read by an accurately calibrated millivoltmeter is 0·1948 volt. We shall also suppose that the temperature of the rod is 35·6° C. and that the current flowing through it and the voltmeter in parallel, as read by a Kelvin balance is 398·9 amperes. If the resistance of the millivoltmeter with its connecting leads be 7·30 ohms the current flowing through it will be 0·1948/7·3, or 0·027 ampere nearly. The current in the conductor may therefore be taken as 398·9 amperes, and hence, the resistance between the two equipotential surfaces passing through the points of contact of the knife edges will be 0·1948/398·9, that is, 0·0004883 ohm. If we assume that the lines of flow of the current are parallel to the axis of the conductor so that the equipotential surfaces are planes perpendicular to this axis, then since 0·7854 is the area of the cross section of the rod, we have

$$\rho_{35 \cdot 6} = 0{\cdot}0004883 \times 0{\cdot}7854/200$$
$$= 1{\cdot}917 \text{ microhms,}$$

and therefore,

$$\rho_{15 \cdot 6} = 1{\cdot}917(1 + 0{\cdot}0042 \times 15{\cdot}6)/(1 + 0{\cdot}0042 \times 35{\cdot}6)$$
$$= 1{\cdot}917/1{\cdot}079$$
$$= 1{\cdot}777.$$

Now at this temperature the standard volume resistivity is 1·731. The percentage conductivity of the copper forming the rod is, therefore, $100 \times 1{\cdot}731/1{\cdot}777$, that is, 97·4. As a 2 per cent. variation from the adopted standard is considered permissible by manufacturers and engineers, this conductor would legally satisfy a specification insisting on a 99 per cent. conductivity but not one insisting on a 100 per cent. conductivity. In practice many tests must be taken, and the conditions of the experiment or the method adopted must be varied in some of the tests, before the experimenter can make certain that his maximum inaccuracy is less than the half of 1 per cent. It has to be remembered that, since the resistance is found by dividing the reading of the millivoltmeter by the reading of the ammeter, the percentage error in the computed resistance is sometimes equal to the sum of the two instrumental percentage errors.

Tinning

Some of the substances, sulphur for instance, in the materials used to insulate copper wires, attack copper. When, therefore, these substances are used the wires are given a coating of pure tin. As the conductivity of tin is less than that of copper, the conductivity of a tinned copper conductor will be slightly less than that of a pure copper conductor of the same diameter. For this reason the conductivity of all tinned copper conductors whose diameters lie between 0·104 and 0·028 inches (No. 12 and No. 22 S.W.G.) inclusive, is allowed to be 1 per cent. lower than that of pure copper.

In measuring a resistance by the fall of potential method, as described in the last section, we assumed that the lines of flow of the current were straight lines. If the metal is not homogeneous or if its diameter vary appreciably, this assumption is not permissible and errors may arise from this cause. For example, in measuring the resistance of the copper bonds used for rails in electric traction the result depends on the equipotential surfaces chosen. This is illustrated in Fig. 1. Here *ab* is a short cylindrical copper conductor connecting two large copper cylinders *A* and *B*. The lines and arrow-heads indicate the direction of the flow of the current through the conduc-

Lines of flow

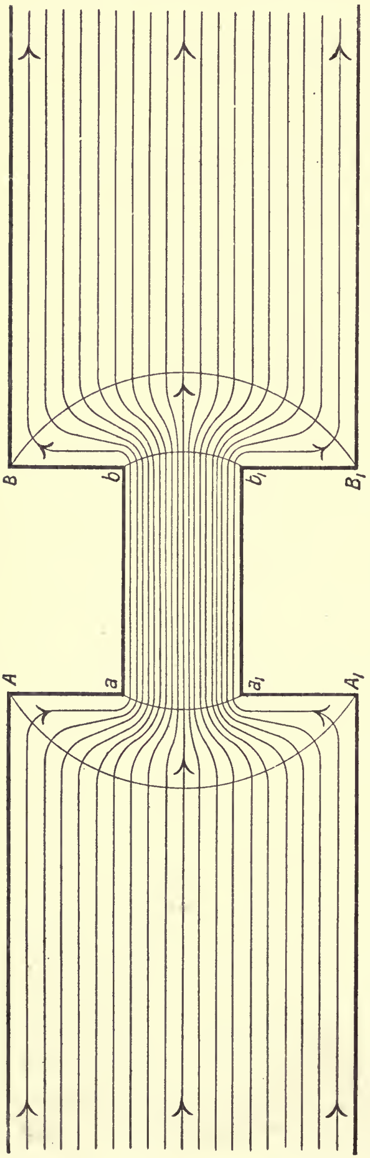

FIG. 1.—Lines of flow of the current when the section of a conductor varies. $A\,A_1$, $a\,a_1$, $B\,B_1$, $b\,b_1$, are sections of the equipotential surfaces through A, a, B and b respectively. These surfaces cut the stream lines at right angles.

tors. The curved lines AA_1, BB_1, aa_1 and bb_1, cutting the lines of flow at right angles are sections of equipotential surfaces by the plane of the paper. If we were to put the knife contacts at A and B, and proceed as in the last section but one, the resistance measured would be that between the surfaces AA_1, and BB_1, and unless these surfaces were accurately known and also the temperatures at the various sections, it would be impossible to deduce the conductivity of the copper. Similarly if the knife edges were placed at a and b we would get another value of R, but as the equipotential surfaces are still appreciably curved an error would be introduced if we made calculations on the assumption that they were planes.

Measuring the rise of temperature

In the report of the Standardization Committee of the American Institution of Electrical Engineers (*Journal*, 1907), it is recommended that the rise of temperature in all conductors should, when practicable, be determined by their increase of resistance. The resistance may be measured either by a Wheatstone's bridge method or preferably by an ammeter and voltmeter. The temperature calculated in this way is usually higher than that obtained by placing a thermometer against the conductor. It is also recommended that, when a thermometer is placed against the surface of the object of which the temperature is being measured, the bulb should be covered by a pad of definite area. For instance, a convenient pad may be made of cotton waste contained in a shallow circular box about $1\frac{1}{2}$ in. in diameter. The bulb of the thermometer is inserted through a hole in the side of the box. If the pad be too large it interferes with the natural radiation of heat from the metal surface and thus introduces complications into the test.

The formula

$$R = R_0(1 + 0{\cdot}0042t) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1),$$

which shows the connexion between the resistance R of copper at $t°$ C., and its resistance R_0 at 0° C., enables us to calculate the temperature rise when the values of the initial and final resistances, R and R', are known. If R' be the resistance when the temperature is $t+x$, we have by (1)

$$R' = R_0\{1 + 0{\cdot}0042(t+x)\} \quad \ldots \quad \ldots \quad \ldots \quad (2).$$

Hence from (1) and (2),

$$\begin{aligned} R'/R &= \{1+0{\cdot}0042(t+x)\}/\{1+0{\cdot}0042t\} \\ &= 1 + 42x/(10{,}000+42t), \end{aligned}$$

and therefore, $x = (238+t)(R'/R-1) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3).$

As an example of the use of (3), let us suppose that R is 81·8 ohms at the initial temperature of 12° C., and that the resistance is finally 85·8 ohms. By (3), we have

$$\begin{aligned} x &= (238+12)(85{\cdot}8/81{\cdot}8-1) \\ &= 1{,}000/81{\cdot}8 \\ &= 12{\cdot}2°\ \text{C}. \end{aligned}$$

As another example let us take the case of an armature winding. Before the test let us suppose that its resistance was 0·230 of an ohm and that the temperature was 25° C., and that after carrying a current for some time the resistance rises to 0·271 of an ohm. In this case

$$\begin{aligned} x &= (238+25)(0{\cdot}271/0{\cdot}23-1) \\ &= 46{\cdot}9°\ \text{C}. \end{aligned}$$

Temperature coefficients of metals

J. Dewar and J. A. Fleming (*Phil. Mag.*, [5], vol. 36, p. 271, 1893) give the following values of the mean temperature coefficients of pure metals for temperatures from 0° to 100° C.

Metal	a	Metal	a
Aluminium . . .	0·00435	Nickel	0·00622
Cadmium . . .	0·00419	Palladium . . .	0·00354
Copper	0·00428	Platinum . . .	0·00367
Gold	0·00377	Silver	0·00400
Iron	0·00625	Thallium . . .	0·00398
Lead	0·00411	Tin	0·00440
Magnesium . . .	0·00381	Zinc.	0·00406

It is interesting to notice that the temperature coefficient of platinum is practically the same as the temperature coefficient of the pressure of a gas at constant volume.

Stranded cables

When the area of the cross section of the copper in a cable has to be greater than 6,400 circular mils, that is, than the area of the cross section of a solid cylindrical conductor 0·08 of an inch in diameter (No. 14 S.W.G.) it is customary to form the conductor of several strands of wire. In general there is one central wire and round this wire is a layer of six wires, and after this the number of wires in successive layers increases in arithmetical progression, the common difference being 6. The number of strands, for instance, in the section of the cable shown in Fig. 2 is 1+6+12, that is, 19.

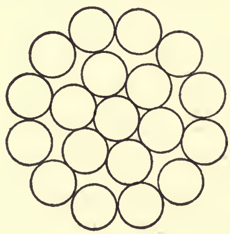

FIG. 2.—Nineteen strand cable.

When there are n layers, the total number N of strands is given by

$$N = 1+6+12+18+ \ldots +6n$$
$$= 3n(n+1)+1.$$

Thus

$$N/3 = n^2+n+1/3$$
$$= (n+1)^2 - n - 2/3$$

Hence $(N/3)^{1/2}$ is greater than n but less than $n+1$. Consequently the number of layers in a cable of N strands is the integral part of $(N/3)^{1/2}$. For example if N were 331, the number of layers would be 10, for $(N/3)^{1/2}$ equals $(110{\cdot}3\,..)^{1/2}$, and the integral part of this radical is obviously 10. We should therefore have ten layers containing 6, 12, 18 .. 60 wires in addition to the central wire.

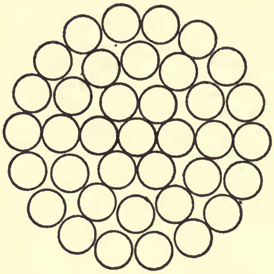

Fig. 3.—Cross section of a cable containing 37 strands of wire. The middle wire is straight, and consecutive layers are spiralled in opposite directions.

In ordinary cables the number of strands used are 1, 7, 19, 37, 61, 91 or 127. A cable consisting of N strands of No. M wire is called an N/M cable.

In Fig. 3 the cross section of a cable consisting of 37 strands is shown. It will be noticed that after the first layer the sections of the strands do not necessarily touch the sections adjacent to them. In practice, consecutive layers of the strands are given a slight twist in opposite directions, the effect being that the centres of the sections of the strands in each layer lie on a circle concentric with the section of the central wire. Since the wires in the layers are helical, their sections by a plane perpendicular to the axis of the cable will not be exactly circular.

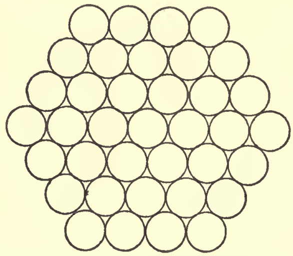

Fig. 4.—Cross section of a stranded cable of 37 wires when the wires are all parallel. Notice that the difference between the numbers of wires in consecutive layers is six.

If all the strands were parallel circular cylinders and if

the cable had to be as compact as possible the section would be hexagonal in shape (Fig. 4), and every conductor inside the outside layer would touch the six adjacent conductors.

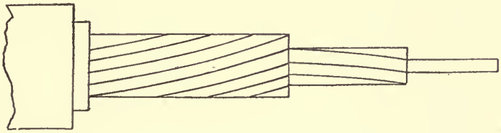

FIG. 5.—Stranded cable. The strands in successive layers are spiralled in opposite directions.

The effect of giving a helical form to the layers is to make them bind together. The inner and outer boundaries of the layers (Fig. 5) touch concentric cylindrical surfaces. The radius of the inner cylindrical surface which every wire on the nth layer touches is $(2n—1)r$.

We shall now consider the number of strands it would be possible to get on the nth layer on the assumption that the sections of the strands are circles. In Fig. 6, let r be

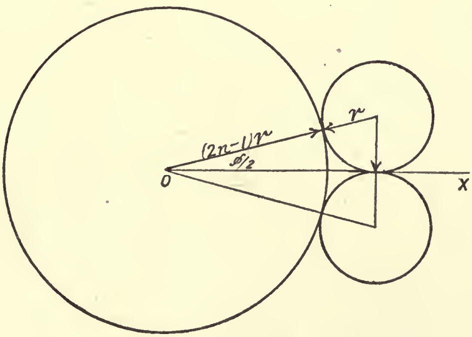

FIG. 6.

the radius of each of the small circles which touch one another and the large circle. Let the radius of the large circle be $(2n—1)r$. The angle ϕ subtended at the centre of

the large circle by the line joining the centres of the two small ones is given by

$$\sin(\phi/2) = r/2nr = 1/2n.$$

Thus the number of the wires that would go round the large circle is the integral part of the function,

$$180/\sin^{-1}(1/2n),$$

the angle $\sin^{-1}(1/2n)$ being measured in degrees. The values of this function for various values of n are given in the following table.

n	1	2	3	4	5	6	7
Function. .	6	12·4	18·8	25·1	31·4	37·7	43·98

The numbers are approximately in arithmetical progression, the common difference being very nearly 2π or 6·28 . . , when n is greater than 2. Thus theoretically we could put 6, 12, 18, 25, 31 . . wires in the successive layers instead of the 6, 12, 18, 24, 30 . . used in practice.

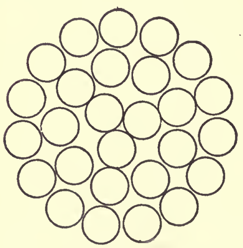

FIG. 7.—Section of a 27 strand cable having a central core of three strands.

Cables are occasionally made having a central core formed of three strands (Fig. 7), or more rarely of four strands. In the case when the core has three strands, the number of strands in the nth layer is $6n+3$, and the total number N of strands is given by

$$N = 3+9+15+ \ . \ . \ . \ +6n+3$$
$$= 3(n+1)^2.$$

Thus the number of layers is one less than $\sqrt{N/3}$.

Proceeding as before, we find that the number of wires in the nth layer is the integral part of the function

$$\frac{180}{\operatorname{Sin}^{-1}\{1/(2/\sqrt{3}+2n)\}}$$

where the angle is expressed in degrees.

The values of this function are given in the following table. It will be seen that when n is greater than 3 the

n	1	2	3	4	5	6
Function . .	9·7	16·1	22·5	28·8	35·1	41·4

numbers practically form an arithmetical progression, the common difference being approximately 2π or 6·28. . . Theoretically, therefore, it is possible to put 16 instead of 15 wires in the second layer and 35 instead of 33 on the fifth.

Effect of lay on the mass of the conductor

When a stranded cable has a central wire the axis of this wire is a straight line but the axis of every other wire of the cable forms a helix, all the helices forming a layer having practically the same pitch. If a point moving along the helical axis of a strand of the cable make a complete revolution round the central wire when it has advanced a distance parallel to this wire equal to n times the diameter of the helix, the wire is said to have a lay of 1 in n. If a be the angle which the tangent LP at any point L of the helix (Fig. 8) makes with a line through L parallel to the central wire, if $LN=nd=l$, where d is the diameter of the helix, and if NP be at right angles to LN, then LP will be the length x of the helical wire corresponding to a length l of the central wire, and PN will equal πd. Hence $\tan a=\pi d/l=\pi/n$, and $x=l\cdot\sec a=l(1+\pi^2/n^2)^{1/2}$. Since l is the pitch of the

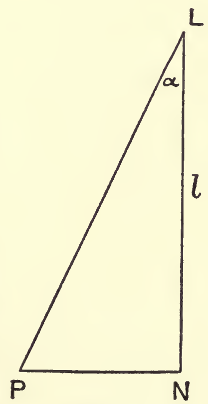

FIG. 8.

helix and equals nd, we see that the pitch of the helical strands in the various layers varies as the diameters of these layers, provided that the lay is the same for all the strands.

We shall now consider the effect of the lay on the mass of the copper required. Let us take the case of a 7-strand cable of length l and let us find the factor by which the mass of the central wire has to be multiplied in order to get the mass of the whole copper in the length l. By the formula given above the length of the six helical wires in the first layer is $l(1+\pi^2/n^2)^{1/2}$. Hence the required multiplier is $1+6(1+\pi^2/n^2)^{1/2}$.

For example, if the lay were 1 in 20, n would be 20 and the multiplier would be 7·0736. If the wires were straight the multiplier would be 7, and thus, the effect of the lay is to increase the mass of the conductor by a little more than 1 per cent.

Some manufacturers use a lay as low as 1 in 12 and others as high as 1 in 30. The value usually taken is 1 in 20.

In the following table the factors for multiplying the mass of a strand equal in length to the cable in order to get the mass of the conductor are given for lays of 1 in 12, 1 in 20 and 1 in 30.

No. of Strands	Multiplier		
	Lay of 1 in 12	Lay of 1 in 20	Lay of 1 in 30
3 . . .	3·101	3·037	3·016
4 . . .	4·135	4·049	4·022
7 . . .	7·202	7·074	7·033
12 . . .	12·404	12·147	12·066
19 . . .	19·607	19·221	19·098
37 . . .	38·213	37·661	37·198
61 . . .	63·022	61·736	61·328
91 . . .	94·033	92·103	91·492

The cable with twelve strands has a core of three strands.

Effect of lay on the resistance of the conductor

Let us now consider how the resistance of a stranded cable varies with the lay of the wires. As the greater the lay of the wires, the greater the mass of the conductor in a given length of the cable, it might at first sight be thought that the resistance would diminish as the lay is increased. If we remember, however, that the great bulk of the lines of flow of the current in the strands must follow helical paths we should expect that the resistance of all these paths in parallel will be greater than the resistance of the shorter paths when the strands are straight and this is found to be the case in practice.

In a 7-strand cable, for example, if r be the resistance of the central strand, the resistance of the other six strands in parallel will be $r(1+\pi^2/n^2)^{1/2}/6$, where n is the lay of the cable, assuming that there is no flow of current from one strand to another. Hence the factor by which the resistance of the central wire has to be multiplied by in order to get the resistance of the cable is $1/\{1+6/(1+\pi^2/n^2)^{1/2}\}$, and this is always greater than one-seventh. Hence the effect of the twisting is to increase the resistance of the cable per unit-length.

In the following table the factors for multiplying the resistance of a single strand equal in length to the cable in order to get the resistance of the cable are given for lays of 1 in 12, 1 in 20, and 1 in 30.

No. of Strands	Multiplier		
	Lay of 1 in 12	Lay of 1 in 20	Lay of 1 in 30
3	0·34457	0·33742	0·33516
4	0·25843	0·25307	0·25137
7	0·14696	0·14436	0·14353
12	0·08614	0·08436	0·08379
19	0·05431	0·05324	0·05290
37	0·02791	0·02735	0·02717
61	0·01694	0·01659	0·01648
91	0·01136	0·01112	0·01105

Permissible current in cables

In the wiring rules (1907) of the English Institution of Electrical Engineers the following table of the maximum permissible currents for copper conductors laid in casing or tubing is given. The maximum currents may be calculated from the formula

$$C = 2{\cdot}6S^{0{\cdot}82},$$

where C is the current in amperes and S is the sectional area in thousandths of a square inch.

Number and Gauge of Strands. S.W.G. or Ins.	Section Sq. Ins.	Mass Lbs. per 1,000 yds.	Max. Current Amps.	Current Density Amps. per sq. in.	Length in yards for 1 volt drop
3/25	0·00092	11·12	2·5	2,800	15
3/24	0·00112	13·45	2·9	2,600	16
3/23	0·00133	16·01	3·3	2,500	17
1/18	0·00181	20·92	4·2	2,300	18
3/22	0·00181	21·79	4·2	2,300	18
7/25	0·00216	25·87	4·9	2,250	18
3/21	0·00237	28·45	5·3	2,250	19
1/17	0·00246	28·48	5·4	2,200	19
7/24	0·00262	31·29	5·7	2,200	19
3/20	0·00299	36·02	6·4	2,150	19
7/23	0·00311	37·24	6·6	2,150	20
1/16	0·00322	37·20	6·8	2,100	20
3/19	0·00370	44·47	7·6	2,050	20
1/15	0·00407	47·08	8·2	2,000	21
7/22	0·00424	50·70	8·5	2,000	21
1/14	0·00503	58·12	9·8	1,950	21
3/18	0·00532	64·02	10·3	1,950	21
7/21	0·00554	66·21	11·0	1,950	21
7/20	0·00701	83·81	13·0	1,850	22
7/19	0·00865	103·5	15·0	1,750	24
7/18	0·01250	149·0	21·0	1,700	25
7/17	0·017	202·8	27·0	1,600	26
19/20	0·019	228·0	29·0	1,550	27
7/16	0·022	264·8	33·0	1,500	28
19/19	0·023	281·0	35·0	1,450	28

Number and Gauge of Strands. S.W.G. or Ins.	Section Sq. Ins.	Mass Lbs. per 1,000 yds.	Max. Current Amps.	Current Density Amps. per sq. in.	Length in yards for 1 volt drop
7/0·068 in.	0·025	299·0	36·0	1,450	29
7/15	0·028	335·0	40·0	1,450	29
19/18	0·034	405·0	47·0	1,400	30
7/14	0·035	414·0	48·0	1,400	30
19/17	0·046	551·0	60·0	1,300	32
7/0·095 in.	0·050	584·0	65·0	1,300	32
19/0·058 in.	0·050	591·0	65·0	1,300	32
19/16	0·060	720·0	75·0	1,250	33
19/15	0·075	911·0	91·0	1,200	35
19/14	0·094	1125	108·0	1,150	36
19/0·082 in.	0·100	1182	113·0	1,150	36
37/16	0·117	1403	130·0	1,100	37
19/13	0·125	1488	136·0	1,100	38
37/15	0·150	1776	157·0	1,100	39
19/0·101 in.	0·150	1793	155·0	1,050	40
37/14	0·180	2192	187·0	1,050	40
37/0·082 in.	0·20	2303	200·0	1,000	40
37/0·092 in.	0·25	2900	238·0	950	42
37/0·101 in.	0·30	3494	280·0	950	43
37/0·110 in.	0·35	4145	320·0	900	45
61/13	0·40	4781	350·0	900	47
61/0·098 in.	0·45	5425	380·0	850	47
61/0·101 in.	0·50	5762	425·0	850	47
61/0·108 in.	0·55	6588	450·0	800	48
61/0·110 in.	0·60	6836	490·0	800	48
61/0·118 in.	0·65	7865	530·0	800	50
91/0·098 in.	0·70	8094	550·0	800	50
91/0·101 in.	0·75	8597	590·0	800	50
91/12	0·80	9115	625·0	800	51
91/0·110 in.	0·90	10200	670·0	750	53
91/0·118 in.	1·00	11730	750·0	750	54
127/0·101 in.	1·00	12000	750·0	750	55

It is to be noticed that the symbol 19/0·058 in. stands for a conductor of 19 strands of wire, the diameter of

each of which is 0·058 inches. The sizes 3/25, 3/24, 3/23 are the usual sizes of the conductors used in electric light fittings. It is worth remembering that when the current density is 1,000 amperes per square inch the pressure drop is 1 volt for 40 yards. At this current density, for instance, if the going and return conductors are each 40 yards long the difference of pressure between the far ends of the conductors will be 2 volts less than that between the ends where they are joined to the switchboard. The difference in the two values of the pressure is the pressure required owing to the resistance of the conductors. In practice the maximum permissible value of the current in conductors is fixed by the voltage drop, and not by the rise of temperature of the conductor. By the Board of Trade Rules, the pressure at any consumer's terminals must not vary by more than 4 per cent. from the declared constant pressure, and this regulation generally necessitates a low current density in the mains.

The resistance of cables

In the following table, compiled from the wiring rules (1907) of the Institution of Electrical Engineers the resistances at 60° F. of copper conductors per 1,000 yards are given in ohms.

Gauge S.W.G. or ins.	Res.	Gauge S.W.G. or ins.	Res.	Gauge S.W.G. or ins.	Res.
3/25	26·01	7/23	7·721	7/18	1·930
3/24	21·50	1/16	7·473	7/17	1·418
3/23	18·07	3/19	6·504	19/20	1·267
1/18	13·29	1/15	5·905	7/16	1·086
3/22	13·27	7/22	5·672	19/19	1·026
7/25	11·12	1/14	4·783	7/0·068 in.	0·962
3/21	10·16	3/18	4·516	7/15	0·858
1/17	9·761	7/21	4·343	19/18	0·713
7/24	9·190	7/20	3·431	7/14	0·695
3/20	8·029	7/19	2·779	19/17	0·523

Gauge S.W.G. or ins.	Res.	Gauge S.W.G. or ins.	Res.	Gauge S.W.G. or ins.	Res.
7/0·095in.	0·493	19/0·101	0·161	61/0·108 in.	0·044
19/0·058 in.	0·488	37/14	0·132	61/0·110 in.	0·042
19/16	0·401	37/0·082 in.	0·125	61/0·118 in.	0·037
19/15	0·317	37/0·092 in.	0·100	91/0·098 in.	0·036
19/14	0·257	37/0·101 in.	0·083	91/0·101 in.	0·034
19/0·082in.	0·244	37/0·110 in.	0·070	91/12	0·032
37/16	0·206	61/13	0·061	91/0·110 in.	0·028
19/13	0·194	61/0·098 in.	0·053	91/0·118 in.	0·025
37/15	0·163	61/0·101 in.	0·050	127/0·101 in.	0·024

High frequency alternating currents

With alternating currents having a frequency not greater than 50 and with conductors the diameters of which are not greater than a centimetre, the above formulae can be used for calculating the resistance. When, however, the frequency of the alternations is high or the diameter of the conductor is large, the effective resistance to the alternating currents is greater than for direct currents. The reason for this is that the current starts at the surface of the wire and takes time to penetrate into the interior.

Let us consider, for example, the currents in a concentric main, that is, a main formed by a solid or a hollow copper cylinder surrounded by a cylindrical tube. With high frequencies the going and return currents distribute themselves in such a way that practically no magnetic forces are produced in the solid copper. There is a concentration of the current on the outside of the inner conductor and on the inside of the outer conductor and this considerably increases the effective resistance of the conductors. Let R and R_a denote the resistances of the inner conductor of a concentric main to direct and to alternating currents respectively. If W is the power expended on the inner conductor when a current A of the given frequency is

flowing in it, R_a equals W/A^2. The value of R_a/R can be found from the following table which is practically the same as that first given by Lord Kelvin. In this table f denotes the frequency, in cycles per second, a the radius of the conductor in centimetres, and ρ its volume resistivity, in C.G.S. measure, so that ρ is 1,000 times the value of the volume resistivity in microhms.

Value of $2\pi a\sqrt{2f/\rho}$	R_a/R
0·0	1·000
0·5	1·000
1·0	1·005
1·5	1·026
2·0	1·078
2·5	1·175
3·0	1·318
3·5	1·492
4·0	1·678
4·5	1·863
5·0	2·043
5·5	2·219
6·0	2·393
7·0	2·743
8·0	3·096
9·0	3·447
10·0	3·798
15·0	5·562
20·0	7·328
30·0	10·86
n·0	0·3535 n approx.

for values of n greater than 30.

When n is greater than 30, $R_a/R = n/2\sqrt{2} = n \times 0{\cdot}3535$ approximately.

$$\text{Hence } R_a = (n/2\sqrt{2})\,(\rho l/\pi a^2)$$
$$= \sqrt{f\rho}\,l/a$$
$$= \rho l/[2\pi a\{(1/2\pi)\sqrt{\rho/f}\}].$$

We see therefore that the value of R_a is the same as if the current were uniformly distributed over a thin skin on the surface of the conductor of thickness $(1/2\pi)\sqrt{\rho/f}$. We see also that with very high frequencies the thicker the

wire the less is the resistance per unit length it offers to alternating currents, but whilst with direct currents the resistance varies inversely as the square of the diameter of the wire, with high frequency alternating currents it varies inversely as the diameter.

Data for calculations

In calculations in connexion with copper cables, the following data will be found useful.

Standard Annealed High Conductivity Copper at 60° *F.*

Volume resistivity (cubic cm.) = 1·696 microhms.

" " (cubic inch) = 0·6679 "

Resistance per mile = $0{\cdot}04232/S$ ohms, where S is the area of the cross section in square inches.

Resistance per yard = $0{\cdot}00002404/S$ ohms.

Mass resistivity = 0·1508.

Resistance per mil foot = 10·20 ohms.

Mass per mile in lbs. = $20350\ S$.

Mass per yard in lbs. = $11{\cdot}56\ S$.

Standard Hard-drawn High Conductivity Copper at 60°*F.*

Volume resistivity (cubic cm.) = 1·731.

" " (cubic inch) = 0·6813.

Resistance per mile = $0{\cdot}04317/S$, where S is the area of the cross section in square inches.

Resistance per yard = $0{\cdot}00002453/S$ ohms.

Mass Resistivity = 0·1539.

Resistance per mil foot = 10·41 ohms.

Mass per mile in lbs. = $20350\ S$.

Mass per yard in lbs. = $11{\cdot}56\ S$.

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INSULATIVITY

CHAPTER III

Insulativity

Insulation resistance—Insulativity—How the insulation resistance of a cable varies with the thickness of the covering—Method of measuring insulation resistance—Price's guard wire—The grade of insulation—Minimum insulation resistance—Methods of Measuring σ—Numerical examples—Rubber—Gutta—References.

Insulation resistance

In order to prevent leakage of current from an electric conductor it is necessary to wrap the conductor in a suitable insulating material. The materials used in practice are cotton, silk, rubber (india-rubber), gutta (gutta-percha), paper and various fibrous materials which are impregnated by oils and waxes of various kinds. For low tension cables the most suitable coverings are those which offer the greatest resistance to a flow of electricity across them, that is, which have the greatest insulation resistance. For high tension work the resistance which the covering offers to a disruptive discharge taking place across it, that is, the electric strength, is the main consideration. Hence the wrappings suitable for low tension cables may not be suitable for high tension cables and vice versa. In this chapter we shall consider low tension cables and the methods of testing them.

Insulativity

We shall define the insulativity σ of a dielectric as the resistance of a centimetre cube of the material to a flow of electric current at right angles to two

opposite faces. It is thus the same as the volume resistivity ρ. It is, however, convenient to use a different symbol as it is customary to measure σ in megohms and ρ in microhms, and hence,

$$\rho = \sigma . 10^{12}.$$

Sir William Preece defines the *specific insulation* σ' of a dielectric as the resistance in megohms of a quadrant cube of the material. Hence we have

$$\sigma' = \sigma . 10^{-9} = \rho . 10^{-21}.$$

How the insulation resistance of a cable varies with the thickness of the covering

In order to understand how the insulation resistance of a cable depends on the thickness of the insulating covering we shall find the insulation resistance of a cable consisting of a copper cylinder covered with a given thickness of homogeneous insulating material of insulativity σ. We shall suppose that the conductor is at potential V, and that the outside of the covering is at potential zero. This would be the case, for instance, if the cable were immersed in water contained in an earthed metal tank. The stream lines of the leakage current will obviously be radial to the cylinder.

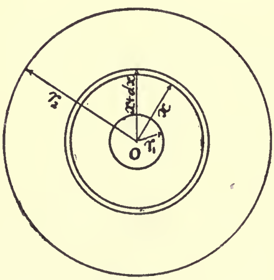

FIG. 9.

Let us imagine that the cylinder of dielectric is divided up into an infinite number of thin concentric cylinders (Fig. 9), the inner and outer radius of one of them being x and $x+dx$ respectively. Consider the flow in a centimetre length of the conductor, that is, the flow from the inside to the outside of the portion of the dielectric contained between two planes each perpendicular to the axis of the cylinder and one centimetre apart. The resistance dR_1 megohms offered to this flow of leak-

age current by the elementary tube of dielectric equals $\sigma dx/2\pi x$, and hence,

$$R_1=\int_{r_1}^{r_2}\frac{\sigma dx}{2\pi x}=\frac{\sigma}{2\pi}\log_\epsilon\frac{r_2}{r_1},$$

where r_1 is the radius of the copper cylinder, and r_2 is the outer radius of the insulating covering. If R be the insulation resistance of a length l cms. of the conductor, we have

$$R=R_1/l=(\sigma/2\pi l)\log_\epsilon(r_2/r_1),$$

for the flow of current across l cms. is obviously l times the flow across 1 centimetre, and its resistance, therefore, is only the lth part of the resistance of 1 centimetre.

We see from the formula that if r_1 is to be kept constant and we wish to increase the insulation resistance n times, the new outer radius of the dielectric would have to be equal to $r_1\,(r_2/r_1)^n$, and hence the thickness of the dielectric would have to be increased from r_2-r_1 to $(r_2{}^n-r_1{}^n)/r_1{}^{n-1}$. The ratio of the new thickness to the old would therefore be $1+r_2/r_1+r_2{}^2/r_1{}^2+\ldots+r_2{}^{n-1}/r_1{}^{n-1}$, and this is much greater than n, except when r_2/r_1 is nearly equal to unity. In the same way, if we keep r_2 constant and diminish r_1 until the insulation resistance is n times as great, we find that the ratio of the new thickness to the old is $1+r_1/r_2+r_1{}^2/r_2{}^2+\ldots+r_1{}^{n-1}/r_2{}^{n-1}$, and this is much smaller than n except when r_1/r_2 is nearly unity. The area of the cross section of the conductor however is diminished in the ratio of $r_1{}^{2n}$ to $r_2{}^{2n}$, and thus, except when n is small, and r_1 and r_2 are not very different, it will be exceedingly small.

The above results illustrate that except when the thickness of the insulating covering is small compared with the diameter of the conductor, increasing the thickness of the covering is not an economical method of increasing the

insulation resistance. As a rule, using a material having n times the insulativity is much preferable to increasing the thickness of the insulation n times. For instance, if n were 4, and r_2/r_1 were 2, the insulation resistance in the former case would be increased four times whilst in the latter it would be increased only 2·32 times.

If the insulating wrappings round a wire may be regarded as concentric cylinders, each cylinder being of homogeneous material, then since the resistances of the cylinders to radial flow are in series, the resultant insulation resistance R in megohms is determined by

$$R=(\sigma_1/2\pi l)\log_\epsilon(r_2/r_1)+(\sigma_2/2\pi l)\log_\epsilon(r_3/r_2)+\quad ..$$

where σ_1, σ_2, .. are the insulativities, and r_2, r_3, .. the bounding radii of the various wrappings. This formula shows that the materials ought to be arranged so that the values of σ_1, σ_2, .. are in descending order of magnitude, the material having the greatest insulativity being next the conductor, for the density of the leakage current is a maximum next the conductor and diminishes the farther we move from the axis.

The insulativity of the dielectric varies rapidly with the temperature, but unlike the resistivities of pure metals, it diminishes as the temperature increases. Hence, if we assume that the relation follows the linear law, we must write

$$\sigma_t=\sigma_{t'}\{1-a(t-t')\}.$$

The values of a are very much larger than for metals. Thus for rubber, Messrs. Siemens Bros. & Co. give 0·047 (Centigrade) as the value of a, and for gutta 0·16, when t is 15·6° C. (60° F.).

A. Campbell (*Proc. Roy. Soc. A.*, vol. 78, p. 207) gives the following table to show how the insulativity of dry cellulose varies with the temperature.

Temp. in deg. C.	$\sigma \times 10^{-6}$
25	1,600
30	900
40	330
50	125
60	40
65	20

Hence an increase of 40° C. causes the insulativity to diminish to one-eightieth of its initial value.

As the accurate measurement of the insulation resistance R of low tension cables is of considerable commercial importance we shall describe fully the method which by a general agreement between manufacturers and engineers is now adopted for making the test.

Method of measuring insulation resistance

Let us suppose that the insulation resistance per mile of a coil of 110 yards of 7/18 cable has to be found. The coil must first be immersed in water, the ends being kept dry, at 60° F. for 24 hours previous to the test. The ends of the coil are next prepared. The tape and protecting material is stripped off the rubber for about 6 inches (Fig. 10) from the ends of the cable. The rubber is then stripped from the conductor for about 3 inches, care being taken that the portion of the rubber left on is intact.

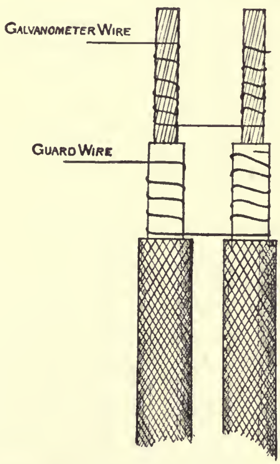

FIG. 10.—The ends of the cable under test.

The voltage of the testing battery is generally taken 50 per cent. greater than that to which the cable will be subjected in actual working. For instance, if it is to be installed in a building supplied from direct current mains, having 220 volts between adjacent mains or 440 volts between the outers, the testing pressure ought to be 660 volts, as in practical work some of the wires will sometimes have to withstand a pressure of 440 volts to earth.

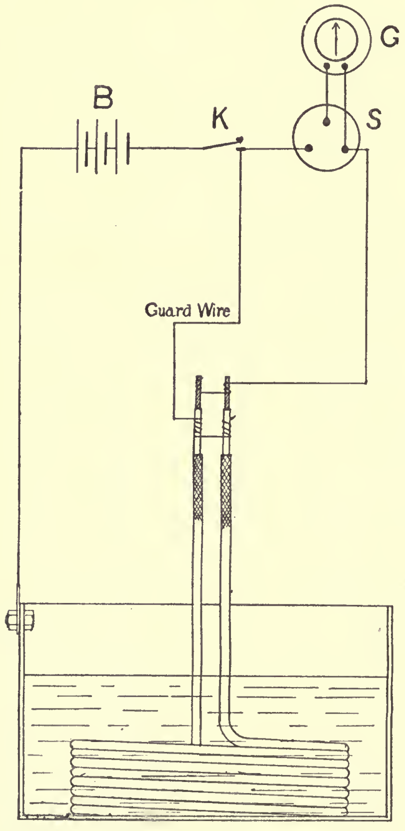

Fig. 11.—Connexions for testing the insulation resistance of a coil of cable.

The battery, galvanometer, and cable are connected in series as shown in Fig. 11. When the key K is closed there will be a deflection of the ray of light reflected from the mirror of the galvanometer, provided that the current through the galvanometer is sufficiently large. The current leaving the battery flows through the water and the insulating covering of the cable to the copper core, and then, through the shunt S and galvanometer G in parallel, back to the battery. If E be the E.M.F. of the battery, which usually consists of three or four hundred small accumulators, we have

$$C=\frac{E}{B+GS/(G+S)+R}\cdot\frac{S}{G+S},$$

where B, G, S and R are the resistances of the battery, galvanometer, shunt, and insulating covering, respectively, and C is the current flowing in them. Unless the cable be broken down or be covered with very inferior insulating material, $B+GS/(G+S)$ will be negligibly small compared with R. We may therefore write $C=mE/R$ or $R=mE/C$, where $1/m = (G+S)/S =$ the multiplying power of the shunt.

In practice, the current C is seldom even approximately constant, and hence, the deflection varies with the time after closing the switch. As a rule a steady deflection indicates good quality material and a very unsteady one that the insulativity is on the point of breaking down. For electric lighting cables, the convention is made that the deflection is read after one minute's electrification. If the galvanometer be calibrated, we know the value of C corresponding to a given deflection, and as E can be measured accurately by a potentiometer or electrostatic voltmeter, R can be found. Thus, if l be the length of the cable in yards, $(l/1760)R$ is the insulation resistance per mile.

To calibrate the galvanometer we place a divided megohm resistance R, and an accumulator, in series with it. Let us suppose that when the multiplying power of the shunt is $1/m_1$ the deflection is d_1. Then the current $m_1E_1/(B+m_1G+R)$ gives the deflection d_1, and since B is negligibly small compared with R, and m_1, E_1, and G, can be accurately determined, the current C corresponding to a deflection d_1 can be found. The voltage E_1 is best determined by comparing it with the voltage of a standard cadmium cell by a potentiometer method. The E.M.F. of a cadmium cell is by Jaeger and Kahle's formula,

$$1{\cdot}0186 - 0{\cdot}000038\,(t-20) - 0{\cdot}00000065(t-20)^2, \text{ volts,}$$

at $t° C$. By varying R and m_1 other deflections can be found corresponding to known currents. Plotting these on squared paper, and drawing a smooth curve through the points, we get an accurate calibration curve for the galvanometer, so that C for any given deflection can be read off at once.

Price's guard wire In measuring exceedingly high resistances particular care has to be taken to ensure that we are not merely measuring the resistance of the path of some surface leakage current. From Fig. 11, we see that a current may flow from the water along the surface of the insulating covering, and then pass through the galvanometer without passing through the insulating covering. To obviate this source of error, W. A. Price uses a guard wire as shown in Figs. 10 and 11. A piece of "flexible" does excellently for the purpose, the bare end of the flexible being wrapped round the rubber insulation. Practically all the surface leakage current will flow along the guard wire without going through the galvanometer at all. In this case, therefore, the deflection of the galvanometer measures only the current leaking through the insulating covering, and hence, we can find the true value of the insulation resistance.

The grade of insulation Cables are generally divided into "grades" according to their insulation resistance. Cables belonging to the 600 megohm grade, for instance, have an insulation resistance ranging from a minimum of 600 megohms per mile for the largest sizes up to 2,000 megohms per mile, which is the insulation resistance of an insulated No. 18 wire belonging to this grade. Such cables are made of tinned copper conductors insulated with pure and vulcanized rubber, and rubber coated tape, the whole being vulcanized together, and covered with braided cotton and preservative compound. The list price of this grade of cable ranges from

about £10 per mile for 1/22 wire to about £1,100 per mile for 61/12. The corresponding cable of 300 Ω (megohm) grade would be about 5 per cent. cheaper, and for 2,500 megohm grade cable about 10 per cent. dearer.

Institution rules

In the wiring rules (1907) of the English Institution of Electrical Engineers, the dielectrics for cables are divided into two classes. In the first or A class, the dielectrics of the cables are impervious to moisture and only need mechanical protection. Vulcanized rubber, for instance, would belong to this class. In the B class are included those dielectrics like paper or fibre which must be kept perfectly dry. They therefore need to be encased in a waterproof sheath. This generally consists of a soft metal tube, a lead tube for example, drawn closely over the dielectric.

Minimum insulation resistance

In the following table the minimum insulation resistances of vulcanized rubber (class A) cables, in megohms per mile, approved by the Institution of Electrical Engineers are given. These insulation resistances are those of cables of *I.E.E.* 600 and 2,500 megohm grades respectively. The minimum insulation resistances, advisable in practice, when the dielectric is fibre or paper, lead covered, are also given. It has been considered advisable to fix minimum values to the radial thicknesses of the dielectric used in different sized and different grade cables. This latter procedure, however, is open to criticism as the mechanical strength of the different kinds of dielectrics used in practice vary largely and the restriction tends to put them all on the same level. The minimum radial thicknesses are quoted in the table.

The dielectric used must not soften at temperatures lower than 176° Fahr. (80° C.) as otherwise there would be a risk of the conductor gradually sinking in it and ultimately

touching the sheath. It is customary to apply an alternating pressure of 2,000 volts for half an hour between the conductor and the sheath, after the cable has been immersed in water for 24 hours, as this pressure will probably break down any weak part in the dielectric covering. The wave of the applied P.D. must be sine shaped and the frequency of the alternating current should be 50.

TABLE

Gauge	Minimum Insulation Resistance in megohms per mile			Minimum Radial Thickness in mils.		
Diameters of strands are given in S.W.G. or ins.	Vulcanized rubber Class A		Fibre or Paper Class B	Vulcanized Rubber Class A		Fibre or Paper Class B
	Up to 250 volts	Up to 650 volts		Up to 250 volts	Up to 650 volts	
3/25	2,000	5,000	140	34	62	—
3/24	2,000	5,000	140	34	62	—
3/23	2,000	5,000	140	35	62	—
1/18	2,000	5,000	140	35	62	—
3/22	2,000	5,000	140	36	62	—
7/25	2,000	5,000	140	36	62	—
3/21	2,000	5,000	140	38	62	—
1/17	2,000	5,000	140	36	62	—
7/24	2,000	5,000	140	37	62	—
3/20	2,000	5,000	140	38	62	—
7/23	2,000	5,000	140	37	62	—
1/16	2,000	5,000	140	36	62	—
3/19	1,250	4,500	140	39	62	—
1/15	1,250	4,500	140	37	62	—
7/22	1,250	4,500	140	39	62	—
1/14	1,250	4,500	140	38	62	—
3/18	1,250	4,500	140	40	62	—
7/21	1,250	4,500	140	40	62	—
7/20	900	4,000	140	41	62	—
7/19	900	4,000	140	42	62	—
7/18	900	4,000	140	44	62	80
7/17	900	4,000	140	47	62	80

Gauge	Minimum Insulation Resistance in megohms per mile			Minimum Radial Thickness in mils.		
Diameters of strands are given in S.W.G. or ins.	Vulcanized Rubber Class A		Fibre or Paper Class B	Vulcanized Rubber Class A		Fibre or Paper Class B
	Up to 250 volts	Up to 650 volts		Up to 250 volts	Up to 650 volts	
19/20	900	3,500	140	48	62	80
7/16	900	3,500	140	49	62	80
19/19	750	3,500	140	50	62	80
7/0·068 in.	750	3,500	140	51	62	80
7/15	750	3,500	140	52	62	80
19/18	750	3,000	120	54	62	80
7/14	750	3,000	120	54	62	80
19/17	750	3,000	120	58	62	80
7/0·095 in.	750	3,000	120	59	62	80
19/0·058 in.	750	3,000	120	59	62	80
19/16	750	3,000	110	62	66	80
19/15	600	3,000	110	66	66	80
19/14	600	3,000	100	71	71	90
19/0·082 in.	600	3,000	100	71	71	90
37/16	600	3,000	90	76	76	90
19/13	600	3,000	90	76	76	90
37/15	600	3,000	90	80	80	90
19/0·101 in.	600	3,000	90	80	80	90
37/14	600	2,500	90	87	87	90
37/0·082 in.	600	2,500	90	87	87	90
37/0·092 in.	600	2,500	80	94	94	100
37/0·101 in.	600	2,500	80	101	101	100
37/0·110 in.	600	2,500	80	107	107	100
61/13	600	2,500	80	113	113	100
61/0·098 in.	600	2,500	80	121	121	100
61/0·101 in.	600	2,500	80	121	121	100
61/0·108 in.	600	2,500	80	125	125	110
61/0·110 in.	600	2,500	80	125	125	110
61/0·118 in.	600	2,500	80	129	129	110
91/0·098 in.	600	2,500	70	129	129	110
91/0·101 in.	600	2,500	70	133	133	110
91/12	600	2,500	70	133	133	120
91/0·110 in.	600	2,500	70	137	137	120
91/0·118 in.	600	2,500	70	141	141	130
127/0·101 in.	600	2,500	70	141	141	130

In the above table the insulation resistance R_1 is given in megohms per mile. This is the unit customarily employed in England. On the Continent, the insulation resistance R is generally given in megohms per kilometre. As a mile equals 1·609 kilometres, it follows that $R = 1{\cdot}609 R_1$.

Methods of measuring σ

In proving the formulae for insulation resistance given above, it must be noticed that we have made the assumption that the insulating material is of homogeneous substance. In finding the value of σ, therefore, care has to be taken that the sample experimented on is homogeneous. When insulating materials are obtained in thin sheets for testing purposes, the thin sheets are often varnished. As this varnish has usually a higher insulativity than the material in the interior of the sheets, we should expect that the values of σ found by tests on thin sheets would be greater than the values found by tests on thick sheets, and this is found to be the case. When also the dielectric sheets are laid between metal plates, with weights placed on the upper one, the value of the resistance between the sheets is found to vary with the mechanical pressure and with the testing voltage, the resistance being smaller the greater the pressure and the greater the voltage. R. Appleyard has shown that it is possible to get consistent results by testing the sheets between suitable mercury electrodes. His method of testing is as follows. The sheet of dielectric is placed vertically between two flat rings of ebonite faced on each other with soft india rubber. Disks of iron are clamped over each ring and form the jaws of a large ebonite vice. Mercury is poured into the hollow spaces between the iron rings and the dielectric, through holes on the top of each disk. The temperature can be conveniently read by placing a thermometer in the mercury. With this arrangement he found, by experiments on presspahn,

that the dielectric resistance is sensibly the same whatever the testing voltage may be, and that it is practically independent of the time of application of the pressure.

In the following table rough approximations to the value of σ for various insulating materials are given. As the insulativity σ generally varies very rapidly with temperature, the numbers quoted only indicate the order of the magnitude of σ which might reasonably be expected.

Dielectrics	$\sigma \times 10^{-6}$
Mica	84
Gutta	450
Rubber	10,000
Ebonite	30,000
Glass	20,000

When two or three hundred yards of a cylindrical wire cable insulated with a known thickness of the insulating material is available, we can find σ by measuring the insulation resistance of the cable.

For instance, if R_1 is the insulation resistance of the cable in megohms per mile, we have, with our usual notation

$$R_1 = \frac{\sigma}{2\pi l} \log_\epsilon \frac{r_2}{r_1},$$

where l is the number (160,900) of centimetres in a mile. Hence

$$\sigma = \frac{2\pi(160{,}900) \times 0{\cdot}4343 R_1}{\log_{10}(r_2/r_1)}$$
$$= 439\ 000\ R_1/\log_{10}(r_2/r_1)$$
$$= 272\ 900\ R/\log_{10}(r_2/r_1),$$

where R is the insulation resistance in megohms per kilometre. We can thus by finding R_1 or R, and r_1 and r_2 determine σ.

Numerical examples

Example I. In the Ferranti concentric main, which connected the generating station at

Deptford with the distributing station at Trafalgar Square, the insulating material consisted of brown paper and black wax. The insulation resistance per mile after it was laid was found to be 720 megohms. The outer radius of the inner conductor was 0·406 inches, and the inner radius of the outer conductor was 0·922 inches. Hence, by our formula

$$\sigma = 439\ 000 \times 720/\log(922/406)$$
$$= 887{\cdot}6 \times 10^6$$

It is therefore better than gutta but inferior to rubber.

Example II. The insulation resistance of a mile of cable is 1,000 megohms, the radius of the copper is 0·4 of an inch, and of the insulating covering 0·97 of an inch. What is the average value of the insulativity ?

We have, $\sigma = 439\ 000 \times 1\ 000/\log(97/40)$
$$= 1\ 140 \times 10^6.$$

Example III. If the insulativity be 10×10^6, and the ratio of the outer to the inner radius of the insulating covering of a main be 2, find the insulation resistance of the main in megohms per kilometre.

In this case $R = (10^7/272{\cdot}9)\log 2$
$$= 1\ 100 \text{ megohms nearly.}$$

Example IV. If the inner and outer radii of the insulating covering of a cable with a cylindrical core be 0·2 and 0·3 cm. respectively, and its insulation resistance is 1000 megohms per mile, what would be the insulation resistance of a cable consisting of a copper cylinder 0·5 cms. radius covered with insulating material to a depth of 0·1 cm. ?

From the data given for the first cable, we have

$$1\ 000 = (\sigma/439\ 000)\log(1{\cdot}5)$$

and for the second,

$$R = (\sigma/439\ 000)\log(1{\cdot}2),$$

and therefore, $R = \{\log(1{\cdot}2)/\log(1{\cdot}5)\}1000$
$$= 449{\cdot}8.$$

Rubber The best material for insulating cables is rubber. It is a vegetable product being the coagulated milky juice of various trees and shrubs. It contains a small amount of resinous matter soluble in alcohol. The rubbers obtained from Pará, Ceará, and Madagascar which contain very little resinous material are the most expensive, and those from Guatemala and Africa which contain much more resinous material and are not so suitable for insulating purposes are cheaper. Para rubber, which is generally considered the best, is obtained from a large euphorbiaceous tree about 50 feet high. The exact chemical formula for rubber is not yet known, but carbon and hydrogen are its only constituents. Its specific gravity is about 0·92. It is very hygroscopic, the weight of the moisture absorbed being about 20 per cent. of its own weight. At 0° C. it is rigid and cannot be easily elongated, but it is not brittle. When heated to temperatures less than 100° C. it becomes soft and easily stretched, but at 120° C. it practically loses its power of recovery when stretched. At 200° C. it is a thick viscous liquid, and when heated still more it is converted into hydrocarbons and only a small carbonaceous ash is left.

If rubber be exposed to the effects of atmospheric changes it oxidizes and deteriorates rapidly. For this reason it is hardened and vulcanized by the action of sulphur. About 3 per cent. of sulphur is mixed with the rubber in the vulcanizing chamber at 160° C. The sulphur combines chemically with the rubber forming a fairly soft and elastic material. After being vulcanized (cured) the elasticity of the rubber is greatly increased and it is not hardened by cold or softened by heat.

Ozone attacks and destroys rubber rapidly, hence, if brush discharges are likely to take place, it must not be used in high tension cables. Grease has a deleterious action on

rubber. It darkens its colour and makes it sticky. When a larger percentage of sulphur is mixed with the rubber, and it is subjected for a longer time to the action of heat, we get ebonite (vulcanite).

If there is an appreciable quantity of free sulphur in the vulcanized rubber it will attack the copper conductor. For this reason it is customary to tin the copper wires used in cables (*see* p. 29).

Gutta

Gutta like rubber is the dried milky juice of various trees. The most important is the *Isonandra Gutta*, of the order *Sapotaceae*, found in the Malayan Archipelago. Unlike rubber it is practically inelastic, and as it softens at a low temperature it is little used for insulating electric light cables. When insulating wires with gutta, they are first passed through a bath of Chatterton's compound, and are then passed through a press heated so that the gutta is in the liquid state. The gutta is forced out round the wire as it leaves the die. It next passes through cold water to stiffen it. A second or third coating can then be put on in a similar manner.

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DISTRIBUTING NETWORKS

CHAPTER IV

Distributing Networks

Kelvin's law—Distributing networks—Copper mains—Distributing centre—Example—The economy of high pressure—Uniformly distributed load—Excessive current density—Main with a branch circuit—Sections of the mains all different—Numerical example—Feeding from both ends—Single tapping—Two centres feeding a distributing centre and a branch main—Loop fed from one centre—Loop with several feeding centres—Ring main with n feeding points—The proper site of the power station—Example—The feeding centre for a straight main—Practical rule—Booster—The economy of a booster—References.

Kelvin's law

ASSUMING that the generating voltage has been fixed, let us consider the problem of transmitting a definite amount of electrical energy by means of a direct current of given magnitude C from one station to another. If R be the total resistance of the mains for the outgoing and the return current, the power expended in heating them will be C^2R, and since R is inversely proportional to the area x of the cross section of the main used, we see that the annual cost of the energy expended in heating the conductors may be written in the form μ/x, where μ is a constant depending on the cost at which power can be generated, the character of the load, etc. By increasing x we diminish the annual cost of the power that would be expended in heating the mains, but we increase the initial cost of the mains, and therefore, the annual sum that has to be expended in interest on the capital borrowed, and

laid aside for the depreciation in the value of the mains. It is obvious, therefore, that the most economical conductor to choose is the one for which the annual interest and depreciation on the initial cost together with the annual cost of the energy wasted is a minimum. In practice, therefore, we should have to find the sum of these annual charges for cables of the various sizes given in manufacturers' catalogues and choose the cable for which this sum is a minimum.

In the particular case where the interest and depreciation on the initial cost of the cable is proportional to the weight of the conducting material used, and therefore proportional to x, the section of the required cable can be determined very simply mathematically. In this case the interest and depreciation may be expressed by λx where λ is independent of x. Hence the total annual charge for the mains will be $\lambda x+\mu/x$, that is, $\{(\lambda x)^{1/2}-(\mu/x)^{1/2}\}^2+2(\lambda\mu)^{1/2}$. Since the least possible value of the square of a number is zero, and $2(\lambda\mu)^{1/2}$ is independent of x, we see that the total annual charge is a minimum when $\lambda x=\mu/x$. We thus deduce that the most economical conductor to use is that for which the interest and depreciation on its initial cost equals the annual cost of the energy expended in heating it. This is generally known as Kelvin's law. If the initial cost of the cable be only approximately proportional to the area of the cross section, the rule still gives a useful indication of the probable size of the most economical conductor.

Distributing networks

In England the permissible voltage drop in the mains is determined by the Board of Trade Regulations. The seventh rule (B. 7) reads as follows :—

" 7. *Variation of Pressure at Consumer's Terminals.*—The variation of pressure at any consumer's terminals

shall not under any conditions of the supply which the consumer is entitled to receive, exceed 4 per cent. from the declared constant pressure."

To illustrate how this limitation affects the design of a network, let us first consider the case of a 2 wire distributing system (Fig. 12). At the points A_1, A_2, A_3, . . .

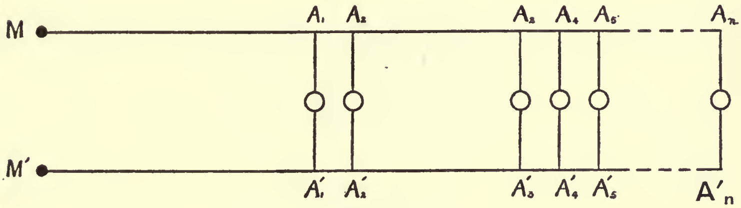

FIG. 12.

let the currents C_1, C_2 ,C_3, . . . be required, and let the distances MA_1, MA_2, . . . from the station terminals be denoted by l_1, l_2, . . . We shall suppose that the section of the mains is uniform throughout and that the potential differences between the points MM', A_1A_1', A_2A_2', . . . are V, V_1, V_2, . . . The portions of the mains MA_1 and $M'A_1'$, are traversed by a current $C_1+C_2+\ldots+C_n$. We have, therefore,

$$V-V_1=(C_1+C_2+\ldots+C_n)\ (2\rho l_1/S),$$

where ρ is the volume resistivity, and S the area of the cross section of the mains. Similarly we find that

$$V_1-V_2=(C_2+C_3+\ldots+C_n)\{2\rho(l_2-l_1)/S\},$$

.. ..

and $$V_{n-1}-V_n=C_n\{2\rho(l_n-l_{n-1})/S\}.$$

Thus the potential difference drop p at the most distant feeding point A_n is given by

$$p=V-V_n=(2\rho/S)(C_1l_1+C_2l_2+\ldots+C_nl_n)$$
$$=(2\rho/S)\Sigma Cl \quad \ldots \quad (1).$$

If $\bar{l}$ denote the distance from M of the centre of

parallel forces equal to C_1, C_2, . . . acting at the points A_1, A_2, . . . when the mains are straight, we have

$$\bar{l}\Sigma C = \Sigma Cl,$$

and thus, from (1),

$$S = (2\rho/p)\bar{l}\Sigma C \quad .. \quad .. \quad .. \quad (2).$$

Hence, when p is given, (2) determines the cross section of the main.

Copper mains

For copper mains, when the temperature is 10° C., $\rho = 1665 \times 10^{-9}$ ohms, and thus (2) becomes

$$S = (3330/p)\bar{l}\Sigma C 10^{-9}.$$

If $\bar{l}$ be measured in metres, and S in square mms., we get

$$S = (333/p)\bar{l}\Sigma C 10^{-4}$$
$$= (\bar{l}/30p)\Sigma C \quad .. \quad .. \quad .. \quad .. \quad (3),$$

approximately. This formula is often used by French electricians and is convenient in practice.

When making calculations instead of showing both mains in the diagram it is sufficient to show one only (Fig. 13), since, in practice, we may regard the return main as identical with the outgoing main.

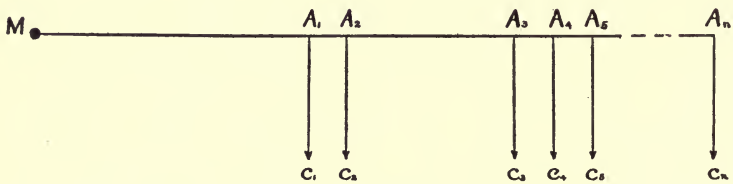

FIG. 13.

Distributing centre

If G be the point in the main MA_n, at which, if all the current ΣC were taken, the voltage drop between M and A_n would be the same as in the actual case, G is called the distributing centre of the load. If we suppose that the main is stretched straight

and that weights equal to C_1, C_2, . . . C_n are placed at A_1, A_2, . . . A_n, respectively, G will be the centre of gravity of these weights. Hence we can use the ordinary statical formula

$$\bar{l}\Sigma C = C_1 l_1 + C_2 l_2 + \ldots + C_n l_n,$$

to determine the length $\bar{l}$ of MG.

Example

Let us suppose that there are five distributing points each 25 metres apart, and that the distance of the first distributing point from the station is 50 metres. Let the currents required at A_1, A_2, . . . A_5, be 5, 10, 30, 10 and 5 respectively. A_3 is obviously the centre of gravity and thus $\bar{l}=100$. Hence substituting in (3), we find that

$$S = 100 \times 60/30p = 200/p \text{ sq. mms.}$$

The economy of high pressure

From (3) we have

$$p = (\bar{l}/30S)\Sigma C \quad \ldots \quad \ldots \quad \ldots \quad (4).$$

If we increase the pressure of supply n times the permissible value of p is generally increased n times also, as the Board of Trade rule fixes the percentage variation of the pressure, and not its absolute magnitude. We see, therefore, from (4) that with the same mains we can supply n times the current. But we have also increased the pressure n times, and hence the load we can supply with the same mains is increased n^2 times. If, for example, we increase the pressure from 100 to 250 volts, we can increase the maximum permissible load $(250/100)^2$, that is, 6·25 times. For this reason it is economical to supply at the highest permissible pressure.

Uniformly distributed load

Let us now suppose that the currents are taken from points A_1, A_2, A_3, . . . A_n (Fig. 14) at equal distances apart and that the main is fed from M. Let us also suppose that the currents are all equal to c, that $MA_1 = a$, and that $A_1A_2 = A_2A_3 = \ldots$

$=x$. The distributing centre of the currents is at a distance $(n-1)x/2$ from A_1, and thus $\bar{l}=MG=a+(n-1)x/2$, and $\Sigma c=nc=C$.

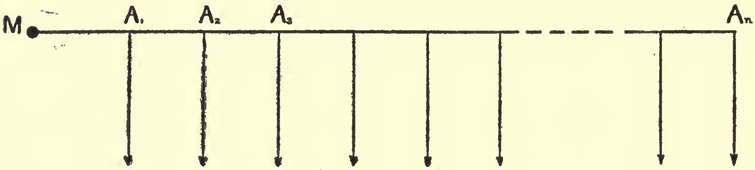

FIG. 14.

Hence, by (2),

$$\begin{aligned}p&=(2\rho/S)\{a+(n-1)x/2\}C\\&=(2\rho/S)[a/2+\{a+(n-1)x\}/2]C\\&=(\rho a/S)C+RC\\&=(R_1+R)C,\end{aligned}$$

where R and R_1 are the resistances of the whole main MA_n, and the part MA_1 respectively. It follows that if the load be uniformly distributed along the main from A_1 to A_n, we have

$$p=(R_1+R)C.$$

If the load had been concentrated at A_n, p would equal $2RC$. Hence, except in the case when MA_1 is negligibly small, the voltage drop with a uniformly distributed load is slightly more than half the value it has when the load is concentrated at the far end.

If H be the power in watts expended in heating the mains, then in the case represented in Fig. 14, we have

$$\begin{aligned}H/2&=(\rho a/S)C^2+(\rho x/S)\{(n-1)^2+(n-2)^2+\ldots+1^2\}(C/n)^2\\&=R_1C^2+\{\rho(n-1)x/S\}\{2-1/n\}C^2/6\\&=R_1C^2+(R-R_1)(2-1/n)C^2/6.\end{aligned}$$

For a load uniformly distributed n is infinite, and thus

$$H/2=(2/3)R_1C^2+(1/3)RC^2.$$

If the load had been concentrated at the far end of the

line, the value of $H/2$ would have been RC^2, and therefore, if MA_1 be small, the power expended in heating the mains when the load is uniformly distributed is very little more than one-third of its value with all the load at the far end.

Feeding from both ends

Let us now consider the case of a main ML (Fig. 15) uniformly loaded and supplied from both ends. If l be the length of the main and C be the total current required, $C/2$ will be the current flowing in at each end, and the greatest permissible voltage drop p will be at the middle of the main. Hence $p=(R/2)\ (C/2)$ where R is the resistance of the whole main,

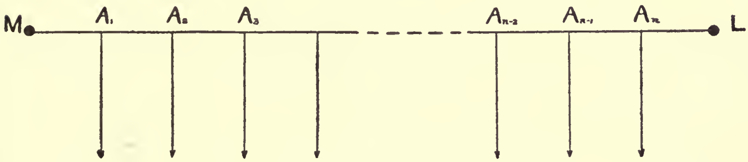

FIG. 15.

and thus $RC=4p$. If the main had been supplied from one end only, the greatest value C' of the current would be given by $RC'=p$. For the same maximum voltage drop, therefore, we could supply four times as much current when we feed from both ends of the main, but the losses in heating the mains would be sixteen times greater in the latter case.

Excessive current density

It sometimes happens that the cross section of the main found by formula (2) makes the current density too high. In this case, the greatest permissible current density is chosen. In only a few cases would it be advisable to choose a current density as high as 2·5 amperes per sq. mm. (approximately 1,600 amperes per sq. in.). Suppose, for

instance, that p is 2, $\bar{l}=20$ metres and $\Sigma C=30$ (Fig. 16). Formula (2) gives

$$S=20\times 30/(30\times 2)=10 \text{ sq. mms.}$$

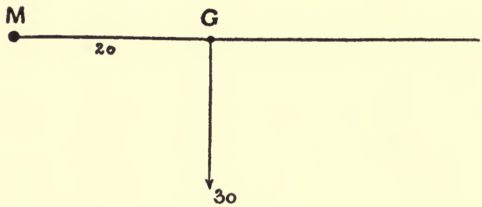

FIG. 16.

This would give a current density of 30/10, that is, 3 amperes per sq. mm. It would be better, therefore, to make the area of the cross section 15 sq. mms. so as to reduce the current density to 30/15, that is, 2 amperes per sq. mm.

Main with a branched circuit

Let us suppose that the main Mab (Fig. 17) has a branch ca joining it at a. Let us also suppose that Mab is the main circuit so that the section of Mab is uniform. Let c_1, c_2, . . . be the

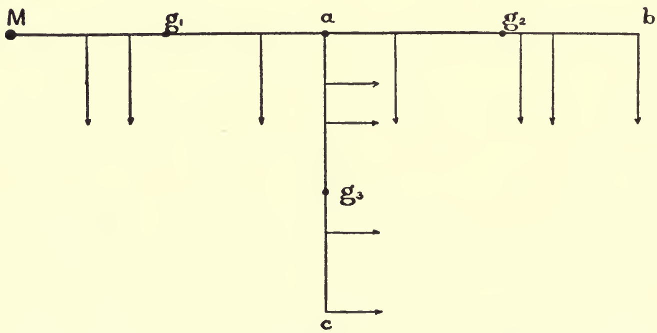

FIG. 17.

currents tapped off between M and a, at distances d_1, d_2, . . . from M. Let c_1', c_2', . . . be the currents tapped

off between a and b, at distances d_1', d_2', . . . from a, and let c_1'', c_2'', . . . be the currents taken between a and c. We see, by (4), that the drop of voltage between a and b is $(ag_2/30S)\Sigma c'$, where g_2 is the distributing centre of the currents c_1', c_2', etc. Similarly if g_1 and g_3 be the distributing centres of the currents c_1, c_2, . . . and c_1'' c_2'', . . . respectively, the voltage drop p between M and b is given by

$$p=\{Mg_1.C+Ma.(C'+C'')+ag_2.C'\}/30S,$$

where C, C' and C'' stand for Σc, $\Sigma c'$ and $\Sigma c''$ respectively. If we write d for Mg_1, d' for ag_2, and l for Ma, this formula becomes

$$p=\{dC+l(C'+C'')+d'C'\}/30S,$$

and hence,

$$S=\{dC+l(C'+C'')+d'C'\}/30p \quad .. \quad .. \quad (5).$$

The voltage drop p_1 from M to a is given by

$$p_1=\{dC+l(C'+C'')\}/30S,$$

and the voltage drop p_2 from a to c by

$$p_2=(d''/30S'')C'',$$

where d'' equals ag_3. Hence, if $p_1+p_2=p$, we must have $p_2=p-p_1$, and therefore,

$$S''=d''C''/30(p-p_1)=S(d''C''/d'C') \quad .. \quad (6).$$

Hence if $d''C''$ be greater than $d'C'$, S'' will be greater than S.

We have now to consider whether it would be more economical to make the section of ab or the section of ac the same as that of Ma. Let V_b denote the volume of the copper required in the first case, and V_c the volume required in the second. If the lengths of ab and ac are l' and l'' respectively, we have

$$V_b/2=S(l+l')+S(d''C''/d'C')l''$$
$$=[\{dC+l(C'+C'')+d'C'\}/30p]\{l+l'+l''(d''C''/d'C')\}.$$

We also have

$V_c/2=[\{dC+l(C'+C'')+d''C''\}/30p]\{l+l''+l'(d'C'/d''C'')\}$, and hence,

$$V_b-V_c=2\{(d'C'-d''C'')/30p\}[l-(l'/d''C''+l''/d'C')\{dC+l(C'+C'')\}].$$

If therefore $(d'C'-d''C'')$ and $[l-(l'/d''C''+l''/d'C')\{dC+l(C'+C'')\}]$ have the same sign, V_b is greater than V_c and thus *ac* should be made the principal branch. If they have not the same sign, *ab* should be made the principal branch.

Sections of the mains all different

Let us now suppose that the sections of the three mains *Ma*, *ab* and *ac* (Fig. 17) are all different but that there is the same voltage drop between *M* and *b* and between *M* and *c*. Let V be the volume of the copper used and x the voltage drop to a. Then using the same notation as in the last section we have

$$V/2=\{dC+l(C'+C'')\}l/30x+d'C'l'/30(p-x)+d''C''l''/30(p-x)=A/x+B(p-x),$$

where

$$A=\{dC+l(C'+C'')\}l/30, \text{ and } B=\{d'C'l'+d''C''l''\}/30.$$

By the differential calculus the rate at which V varies as x increases equals

$$-A/x^2+B/(p-x)^2 \quad \dots \quad \dots \quad \dots \quad (7).$$

This vanishes when $x=p/\{1\pm(B/A)^{1/2}\}$. Since x must be less than p we take the positive sign, and it is easy to see that when $x=p/\{1+(B/A)^{1/2}\}$, V attains its minimum value. This can be seen from first principles as follows. When x is very small the amount of copper used in *Ma* (Fig. 17) must be excessive. As x increases, (7) shows that the volume of copper required is rapidly diminishing. It attains its minimum value when (7) vanishes, and when x is nearly equal to p, the volume required is again very large as the voltage drops in *ab* and *ac* have to be very small.

The rule therefore is to choose the cross sections so that

$$x=p/\{1+(B/A)^{1/2}\}$$
$$=p/[1+(d'C'l'+d''C''l'')^{1/2}/\{dCl+l^2(C'+C'')\}^{1/2}] \quad ..(8).$$

Hence, by (3) we must make

$$\left.\begin{aligned} S&=\{dC+l(C'+C'')\}/30x,\\ S'&=d'C'/30(p-x),\\ \text{and}\quad S''&=d''C''/30(p-x), \end{aligned}\right\} \quad \cdots \quad \cdots \quad \cdots \quad (9)$$

where the value of x is computed from (8).

Numerical example

Let the numerical data of the problem be as given in Fig. 18, so that we have $d=100$, $d'=300$, $d''=80$, $l=250$, $l'=400$, $l''=200$, $C=40$, $C'=15$,

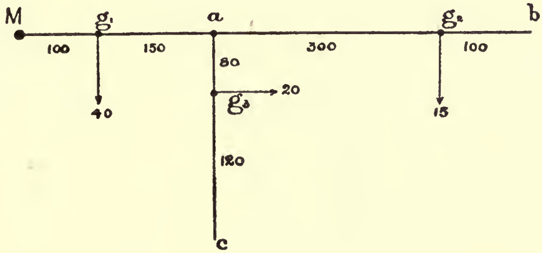

Fig. 18.

$C''=20$, the lengths being measured in metres, and the currents in amperes. Let us also suppose that p is 4 volts, and that Ma and ab are to have the same section S.

We have, therefore, by (5)

$$S=\{100\times 40+250\times 35+300\times 15\}/(30\times 4)$$
$$=143{\cdot}75 \text{ sq. mms.}$$

We have also, by (6), if p is to be the voltage drop from M to c,

$$S''=143{\cdot}75(d''C''/d'C')$$
$$=143{\cdot}75(80\times 20/300\times 15)$$
$$=51{\cdot}1 \text{ sq. mms.}$$

It is interesting to compare these numbers with the

numbers obtained, by (8) and (9), for the most economical solution. From (8), we find that x equals

$$4/[1+(300.15.400+80.20.200)^{1/2}/(100.40.250+250^2.35)^{1/2}],$$

which is nearly equal to 2·2.

Hence, by (9),

$$S=\{100.40+250.35\}/66=193 \text{ approx.}$$
$$S'=300.15/54 \qquad =83{\cdot}3 \quad ,,$$
$$S''=80.20/54 \qquad =29{\cdot}6 \quad ,,$$

The first solution requires 2,074,000 cubic cms. of copper and the second 1,750,000 cubic cms. A saving of about 16 per cent. in the quantity of the copper used could thus be made by adopting the second solution.

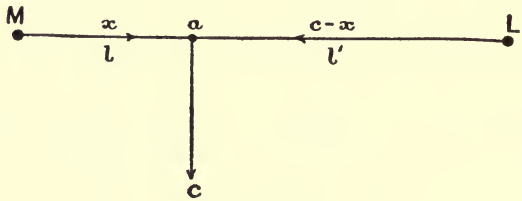

Fig. 19.

Feeding from both ends. Single tapping

Let us suppose that the feeding centres M and L (Fig. 19) are at the same potential, and that the resultant current C branches off at a. If x be the current entering at the feeding centre M, and $C—x$ be the current between L and a, then since the voltage drop p from M to a equals the voltage drop from L to a, we have

$$(\rho l/S)x=(\rho l'/S)\ (C-x)=p,$$

and therefore, $\quad x=Cl'/(l+l').$

We also have,

$$\begin{aligned} S&=xl/30p \\ &=Cll'/\{30p(l+l')\} \\ &=Cl(L-l)/30pL \quad \dots \quad \dots \quad \dots \quad (10), \end{aligned}$$

where $L=l+l'$. Hence S has its maximum value when $l=L/2$.

If $l=L/2$, the currents in l and l' are each equal to $C/2$, and by (10), $S=CL/120p$. If $l=L/4$, x is $3C/4$, $C-x$ is $C/4$, and $S=(12/16)CL/120p$. If $l=L/8$, x is $7C/8$, $C-x$ is $C/8$, and $S=(7/16)\ CL/120p$. By supposing L and M to be coincident, we see that, except in the case when a is midway between M and L, more copper is required when the distributing centre is fed from two centres than when it is fed from the nearer one only. We have, however, an additional security for the continuity of the supply.

Two centres feeding a distributing centre and a branch main

Let us now consider the case of two feeding centres M and L (Fig. 20) supplying a distributing centre at a and a branch main at b. Let the current required at a be C_2 and that

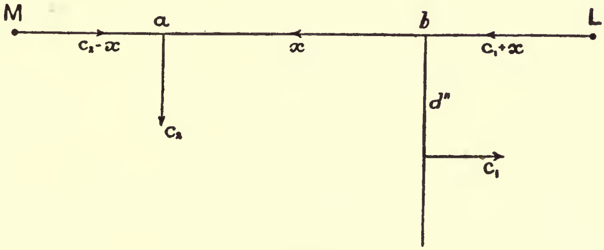

FIG. 20.

at b be C_1. Let also C_2-x be the current in Ma, and C_1+x be the current in Lb. We shall suppose that the section S of the main joining M and L is uniform. Then, since the voltage drop from M to a must be equal to that from L to a, we have, by (4)

$$l(C_2-x)/30S=\{l'x+l''(C_1+x)\}/30S,$$

and thus, $$x=(lC_2-l''C_1)/(l+l'+l'') \quad .. \quad .. \quad (11),$$

where $l=Ma$, $l'=ab$ and $l''=bL$.

From (11) we see that if lC_2 be greater than $l''C_1$, x is

positive, and the direction of the current is as indicated in Fig. 20. When, however, lC_2 is less than $l''C_1$, the current is in the reverse direction. It is also to be noticed that the value of x is independent of the size of the section either of the main or the branch main.

We shall now find the areas of the cross sections of the mains so that the volume of the copper used in them may be a minimum. The size of the submain is determined when the voltage drop p_1 at b is known, for the maximum voltage drop at the end of the submain must not exceed p. Hence if S'' be the area of the cross section of the submain, we have by (3),

$$S''=d''C_1/30(p-p_1) \qquad \ldots \quad \ldots \quad \ldots \quad (12),$$

where d'' is the distance of the distributing centre of the load C_1 from b. By (3), we have

$$S=l''(C_1+x)/30p_1 \qquad \ldots \quad \ldots \quad \ldots \quad (13),$$

and hence the volume of the copper required, namely,

$$2L_1S+2L_2S'',$$

where $L_1=l+l'+l''$, and L_2 is the length of the submain, equals

$$2L_1l''(C_1+x)/30p_1+2L_2d''C_1/30(p-p_1).$$

By the differential calculus this has a maximum or a minimum value when

$$-L_1l''(C_1+x)/30p_1{}^2+L_2d''C_1/30(p-p_1)^2=0,$$

and it is easy to see that when

$$p_1=p/[1+\{L_2d''C_1/L_1l''(C_1+x)\}^{1/2}] \qquad \ldots \quad (14),$$

the volume is a minimum. We can readily find p_1 by this formula, and hence, S and S'' are determined by (13) and (12).

It is to be noticed, however, that the value of p_1 found by (14) may make the voltage drop at a greater than p and this is not permissible. Since the voltage drop at a equals $p_1+p_1l'x/l''(C_1+x)$, we see that this occurs when

$l'x/l''(c_1+x)$ is greater than $\{L_2d''C_1/L_1l''(C_1+x)\}^{1/2}$, or $L_1l'^2x^2$ is greater than $L_2l''d''C_1(C_1+x)$ .. (15). In this case the most economical solution is to choose S, so that

$$S=l(C_2-x)/30p \quad \ldots \quad (16),$$

and also, $$p_1=pl''(C_1+x)/l(C_2-x) \quad \ldots \quad (17).$$

We calculate p_1 by (17), and then S'' is found by (12).

Loop fed from one centre

Let us now consider how to calculate the cross section of a loop of cable (Fig. 21) fed from the centre M. Let the values of the

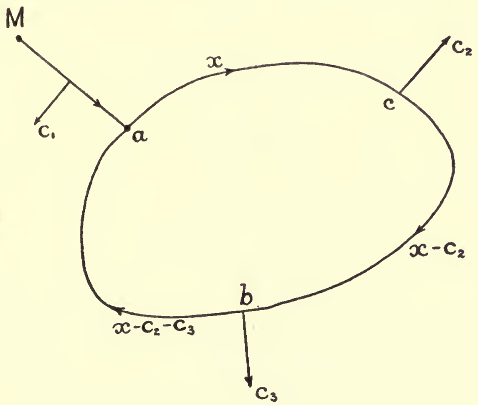

FIG. 21.

currents be as marked in the diagram. We have marked the arrow-heads as if the current were flowing in the same direction all round the loop. This is merely done to obtain algebraical symmetry in our equations. The value of $x-C_2-C_3$, for instance, is always negative. Since the P.D. between a and c added to the P.D.s between c and b, and between b and a must equal zero, we have

$$(\rho/S)\{l_1x+l_2(x-C_2)+l_3(x-C_2-C_3)\}=0,$$

and therefore, $x=\{l_2C_2+l_3(C_2+C_3)\}/(l_1+l_2+l_3)$ .. (18), where l_1, l_2, and l_3, are the lengths of ac, cb, and ba, respectively.

Let us suppose that this value of x is less than C_2. In this case the potential will have its minimum value at C, and the potential drop between M and C will be p. Let p_1 be the P.D. between M and a. Then if S be the section of the main Ma and S' be the section of the cable forming the loop, we have, by (3),

$$\left.\begin{aligned} S &= \{dC_1 + l(C_2 + C_3)\}/30p_1 \\ \text{and} \qquad S' &= l_1 x/30(p - p_1) \end{aligned}\right\} \quad \ldots \quad \ldots \quad (19),$$

where l is the length of the main Ma, and d is the distance of the feeding centre for C_1 from M. Hence if V be the volume of the copper used in the main Ma and in the loop abc, we have

$$\begin{aligned} V/2 &= lS + (l_1 + l_2 + l_3)S' \\ &= \frac{m}{p_1} + \frac{n}{p - p_1}, \end{aligned}$$

by (18) and (19),

$$\left.\begin{aligned} \text{where} \qquad m &= l\{d_1 C_1 + l(C_2 + C_3)\}/30 \\ \text{and} \qquad n &= l_1\{l_2 C_2 + l_3(C_2 + C_3)\}/30 \end{aligned}\right\} \quad \ldots \quad \ldots \quad (20).$$

Now m, n and p are independent of the values of the sections of the mains, and hence by the differential calculus, V will have its extreme values when

$$o = -\frac{m}{p_1{}^2} + \frac{n}{(p - p_1)^2},$$

and when $\qquad p_1 = p/\{1 + \sqrt{n/m}\} \quad \ldots \quad \ldots \quad \ldots \quad (21),$

the volume of the copper employed in the mains has its minimum value. Having found the value of p_1 from (21), the values of S and S' can be readily found from (19).

Loop with several feeding centres

In the loop (Fig. 22) let L, M, O and N be the feeding centres which we suppose are all maintained at the same potential. Let x be the current in Ma, and let currents C_1, C_2 and C_3 be tapped from the loop at points a, b, and c, between M and L. Then, if $Ma = l_1$, $ab = l_2$, $bc = l_3$ and $cL = l_4$, we have

$$xl_1+(x-C_1)l_2+(x-C_1-C_2)l_3+(x-C_1-C_2-C_3)l_4=0,$$

and therefore,

$$x=\{C_1(l_2+l_3+l_4)+C_2(l_3+l_4)+C_3l_4\}/(l_1+l_2+l_3+l_4).$$

If the value of x found from this equation be less than

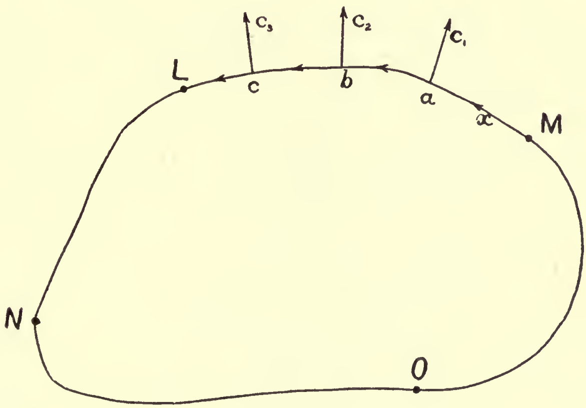

FIG. 22.

C_1, a will be the point of minimum potential. If x be greater than C_1, but less than C_1+C_2, b will be the point of minimum potential and if x be greater than C_1+C_2, c will be the point of lowest potential between L and M. Let us first suppose that x is less than C_1. In this case, by (3), $S=l_1x/30p$. If the value of x lies between C_1 and C_1+C_2, the section of the loop between M and L would be given by

$$S=\{l_1x+l_2(x-C_1)\}/30p,$$

and when the value of x is greater than C_1+C_2, the equation for S is

$$S=l_4(C_1+C_2+C_3-x)/30p.$$

Ring main with n feeding points

We shall now consider the case of a ring main and in order to simplify the formulae we shall suppose that it forms a circle (Fig. 23), with the power station S at its centre, and that the feed-

ing centres are equally spaced round it. We shall also suppose that the load is evenly distributed so that the points of minimum potential are midway between the feeding centres. If there are n feeders, and C is the total

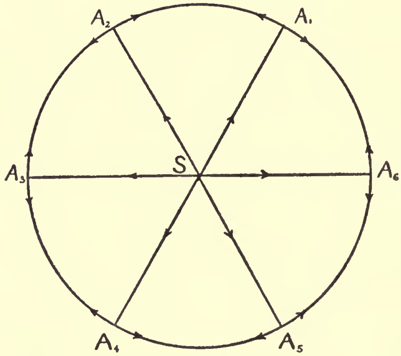

FIG. 23.

current output, C/n will be the current in each feeder and half $(C/2n)$ of this current will flow in one direction round the circle and half in the other.

Let p_1 be the drop of potential from S to any of the feeding points. Then, by (3), the section of each feeder is given in square millimetres by

$$S=(C/n)a/30p_1$$

where a is the radius of the circle in metres.

The section S' of the ring main, in square millimetres, is given by (*see* p. 72)

$$S'=(C/2n)\ (2\pi a/2n)/60(p-p_1).$$

Hence, if V be the volume of the copper required in cubic centimetres, we have

$$V/2=n(C/n)a^2/30p_1+2\pi a(C/2n)(2\pi a/2n)/60(p-p_1),$$
$$=Ca^2/30p_1+Ca^2\pi^2/60n^2(p-p_1)$$

By the differential calculus, V has its minimum value when

$$p_1=np\sqrt{2}/(\pi+n\sqrt{2}).$$

In this case,

$$V/2=(Ca^2/30p)(1+\pi/n\sqrt{2})^2.$$

If n were infinite, the volume V' of the copper required would equal $2Ca^2/30p$, and thus

$$V/V'=(1+\pi/n\sqrt{2})^2.$$

The following table shows how this ratio varies as n increases.

n	1	2	3	4	5	6	7	8	9	10	100
V/V'	10·4	4·46	3·03	2·42	2·09	1·88	1·73	1·63	1·56	1·49	1·04

It will be seen that a substantial saving in copper is effected by increasing the number of the feeders.

The proper site of the power station

When the positions of the feeding centres and the currents they require are fixed, the cost of the feeders varies largely with the site of the generating station. We shall now prove that the most economical site is the "centre of gravity" of the various loads at the various feeding centres. By the centre of gravity of the load is meant the centroid of masses, proportional to the loads at the various feeding centres, placed at these centres.

Let us suppose that $A_1, A_2, \ldots A_n$ (Fig. 24) are the feeding centres, and that $C_1, C_2, \ldots C_n$ are the currents required for them. Then, if p be the maximum permissible voltage drop in the mains between the generating station S and the feeding centres, the section S_1 of the main joining S and A_1 is given by

$$S_1=C_1l_1/30p,$$

and the volume of this main by $2C_1l_1^2/30p$. Hence, if V be the total volume of the copper required

$$V/2=(C_1l_1^2+C_2l_2^2+\ldots+C_nl_n^2)/30p$$
$$=\Sigma Cl^2/30p.$$

Now it is a well-known theorem in statics (*see* Thomson and Tait's *Elements of Nat. Phil.* § 196) that ΣCl^2 has a minimum value when S coincides with the centroid G of masses $C_1, C_2, \ldots C_n$ placed at $A_1, A_2, \ldots A_n$ re-

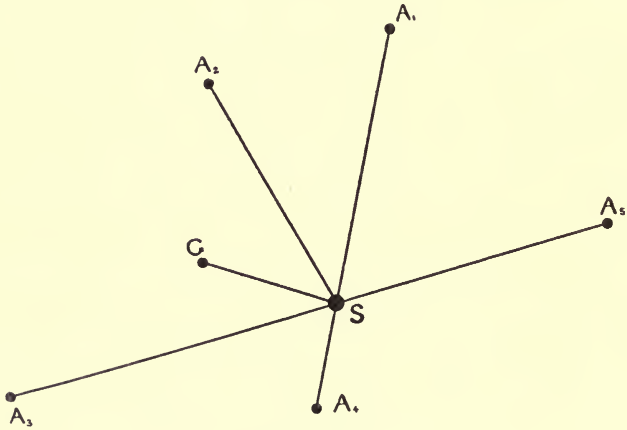

FIG. 24.

spectively. If V_m denote the minimum value of V, it also readily follows that

$$(V-V_m)/2=(C_1+C_2+\ldots+C_n)GS^2/30p,$$

and this is the volume of the copper saved by moving the generating station from S to G.

It is to be noticed that we have chosen the volume of the copper in the two cases so that the power expended in the mains namely $p\Sigma C$ is the same in the two cases.

Example Let us suppose that the feeding centres $A_1, A_2, \ldots A_n$ were equally spaced round a circle of radius a, and that a current C was required at each. Then, if the generating station were at the centre of this circle, the volume of the copper required would be given by

$$V_m/2=nCa^2/30p.$$

If S were at a distance ka from G, the volume V of the copper required would be found from

$$V = V_m + 2nCk^2a^2/30p.$$
$$= V_m(1+k^2).$$

It is therefore very important in practice that k should be small.

The feeding centre for a straight main

Let ML (Fig. 25) be a straight main which we suppose to be uniformly loaded and let A be the position of the generating station. It is required to find the position of a point F in ML, so that

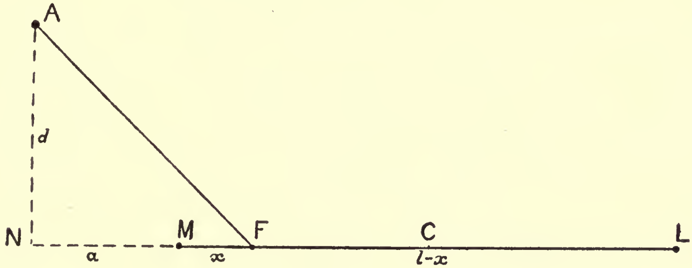

FIG. 25.

when ML is of uniform section, the copper required for the feeder AF and the main ML may be a minimum, subject to the condition that the voltage drop from A to the farthest point of ML must not be less than p. From A (Fig. 25) draw AN at right angles to LM or LM produced. If we take any point F at a distance x from M as the feeding point and if we suppose that x is less than $l/2$, L will be the point of minimum potential. We shall now find the sections of the feeder AF and the main ML so that the copper used in them is a minimum, the voltage drop from A to L being p. Let S_1 be the section of the feeder AF and y its length. Then, by (3),

$$S_1 = Cy/30p_1,$$

where p_1 is the voltage drop between A and F. If we

suppose the main ML to be uniformly loaded, the section S_2 will be given by

$$S_2 = C(l-x)^2/60(p-p_1)l,$$

where l is the length of ML. Hence the volume V of the copper in cubic centimetres is given by

$$V/2 = Cy^2/30p_1 + C(l-x)^2/60(p-p_1).$$

The volume of the copper required, therefore, varies as p_1, and has its extreme values when

$$y^2/p_1{}^2 = (l-x)^2/2(p-p_1)^2,$$

and hence,
$$\sqrt{2}\,y/p_1 = (l-x)/(p-p_1)$$
$$= (l-x+\sqrt{2}\,y)/p$$

the positive sign being taken as this gives the only admissible value, and in this case V has its minimum value V'.

Hence the minimum possible volume V' of copper when the feeding centre is F, is given by

$$V'/2 = (C/60p)\{l-x+\sqrt{2}y\}^2$$
$$= (C/60p)[l-x+\{2d^2+2(a+x)^2\}^{1/2}]^2.$$

We have now to find out what position of F makes this the absolute minimum.

By the differential calculus it follows, almost at once, that when x equals $d-a$, V' is the absolute minimum $V_{min.}$, and hence

$$V_{min.} = (C/30p)(l+a+d)^2.$$

It is to be noticed that in Fig. 27 we have taken a positive when N is to the left of M. If, therefore, N lies between M and C (Fig. 25) at a distance a from M, $MF = d+a$ when the volume has its minimum value $(C/30p)\ (l-a+d)^2$. Now MF cannot be greater than $l/2$ or our assumption that the minimum potential is at L is no longer true. We see, therefore, that if $d-a$, when N is to the left of M, or $d+a$, when N lies between M and C, be not greater than $l/2$ the most economical solution is to make x equal to $d-a$ or $d+a$ according as N is to the left or right of M.

If the given quantities be greater than $l/2$, then, the most economical solution is to make the middle point C of ML the feeding centre. Lastly when N is to the left of M and $d-a$ is negative, M is the proper feeding centre.

Practical rule

From the symmetry of the arrangement when M lies to the right of C similar solutions apply in the various cases. The analytical results lead to the following practical rule for finding the feeding centre for a straight main ML, when the distributing centre is at any point A (Fig. 25). Draw AN at right angles to LM or LM produced. Make the angle NAF equal to 45°, where F lies on LM or LM produced. Then if F lie between N and M, M is the feeding centre, but if it lie between M and C, or on C, F is the feeding centre. Finally, if it lie to the right of C, C is the feeding centre. The following is a graphical illustration of the rule.

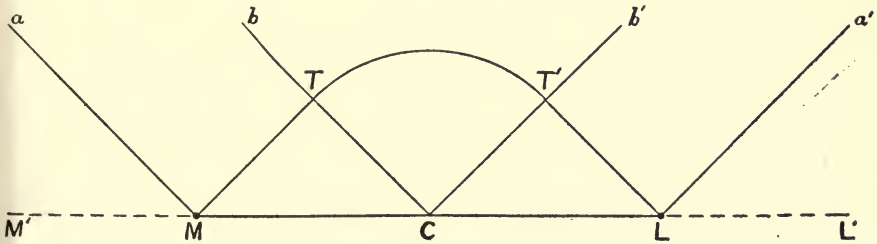

Fig. 26.

Let ML (Fig. 26) be the main which we suppose to be uniformly loaded. Make the angle $M'Ma$ equal to 45° and draw Cb parallel to Ma, where C is the middle point of ML. Similarly make the angle $L'La'$ equal to 45° and draw Cb' parallel to La'. Let us suppose that the generating station A is above the line ML. If A lie within the angle $M'Ma$, M is the feeding centre. If it lie between the parallel lines Ma and Cb, then F is the feeding centre,

where AF is parallel to Ma. If it lie within the right angle bCb', C is the feeding centre. If it lie between Cb' and La' we draw AF parallel to Cb', and finally if it lie within the angle $a'LL'$, L is the feeding centre. An exactly similar solution applies when A is below the line ML.

When the foot of the perpendicular from A on ML falls between M and C' where C' is the middle point of MC, we have

$$V_{min.}=(C/30p)(l+d+a)^2.$$

Now at all points on MT, d equals $-a$, and thus if the generating station A be situated on MT, $V_{min.}$ equals $(C/30p)l^2$ and is therefore constant. If with centre C and radius CT we describe the quadrant TT' of a circle, then, if A be situated at any point on this quadrant, $V_{min.}$ will have the same value. Consequently, if A be situated at any point inside $MTT'LCM$, $V_{min.}$ will be less than if it were situated at M, and if A be situated above $MTT'L$, $V_{min.}$ will be greater than if A were at M.

It is now easy to see that the locus of A for which $V_{min.}$ is constant is a quadrant of a circle between Cb and Cb', a straight line parallel to MT between Cb and Ma, a quadrant of a circle between Ma and TM produced, etc.

Booster

In practice, in order to reduce the initial cost of the copper required when designing a distributing network, it is customary in certain cases to put a "boosting" dynamo or "booster" in series with a feeder, so as to maintain the potential of the distributing centre constant however the load may vary. A booster (Fig. 27) has two directly coupled rotating armatures. One of these is the armature of a shunt wound motor driven from the mains, the other the armature of a series dynamo connected in series with the feeder.

When no current is passing through the dynamo, the

field is practically unexcited and the E.M.F. generated by the rotating armature is negligibly small. When, however, there is a current in the feeder the field magnets

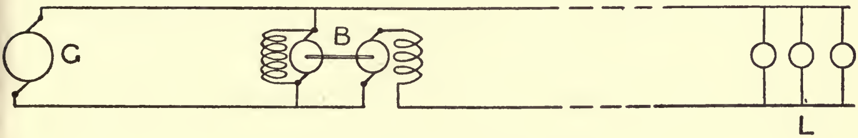

FIG. 27.—Direct current booster.

are excited and an E.M.F. e is generated. If R be the resistance of the dynamo windings and of the outgoing and return feeder, C the current, and E the initial potential of the feeding point, then the new potential will be $E+e-CR$. If the first part of the characteristic of the dynamo be a straight line, it is possible to arrange that $e-CR$ is practically zero for all the values of the current during normal working.

The power expended in the feeding circuit is C^2R and we have now to consider whether it is more economical to use a booster or to increase the weight of the feeder.

The economy of a booster

Let us suppose that the booster is so designed that at full load the drop p of the potential at the distributing centre is the same as if a single feeder of resistance R were used. Let us suppose also that $e=np$. Then, at full load, we have

$$np-CR_1=-p.$$

or $$CR_1=(n+1)p,$$

and thus, $$R_1=(n+1)R.$$

Hence, when a booster is used, the copper required is only the $(n+1)$th part of that required for a feeder main by itself. It has to be remembered that the losses will be $(n+1)$ times greater, but they are only heavy at full load. Hence, for a small distributing centre at a considerable

distance from the station, the use of a booster often effects considerable economies. If the interest saved on the initial cost, by using the booster and the lighter main, be greater than the annual increase of the generating charges together with the cost of the maintenance of the booster, it will be more economical to use a booster. When the distributing centre is large a special dynamo must be used.

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INSULATION RESISTANCE OF HOUSE WIRING

CHAPTER V

Insulation Resistance of House Wiring

Institution Rules—Ohmmeter and Generator—Megger—Electrostatic voltmeter method—Earth lamps—References.

WHEN a building has been wired for the electric light it is necessary to make certain electrical tests to find out whether the mains are properly insulated from one another and from earth. In the wiring rules (1907) issued by the Institution of Electrical Engineers the methods of testing, etc., are described as follows—

Institution rules

"97. The insulation resistance to earth of the whole or any part of the wiring must, when tested previously to the erection of fittings and electroliers, be measured with a pressure not less than twice the intended working pressure, and must not be less in megohms than 30 divided by the number of points under test. For this purpose the points are to be counted as the number of pairs of terminal wires from which it is proposed to take the current, either directly, or by flexibles, to lamps or other appliances.

"98. Current must not be switched on until the following test has been applied to the finished work :—

"The whole of the lamps having been connected to the conductors and all switches and fuses being on, a pressure equal to twice the working pressure must be applied and

the insulation resistance of the whole or any part of the installation must not be less in megohms than 25 divided by the number of lamps. When all lamps and appliances have been removed from the circuit, the insulation resistance between conductors must not be less than 25 megohms divided by the number of lamps. The insulation of any individual sub-circuit must not fall below 1 megohm. Any motor, heater, arc lamp or other appliance may be connected to the supply of electrical energy provided that the insulation of the parts carrying the current measured as above, is greater than 1 megohm from the frame or case.

"99. The value of systematically inspecting and testing apparatus and circuits cannot be too strongly urged. Records should be kept of all tests, so that any gradual deterioration of the system may be detected. Cleanliness of all parts of the apparatus and fittings is essential.

"100. Before making any repairs or alterations, the circuits which are being attended to must be entirely disconnected from the supply."

It is advisable to make two insulation tests between the mains. For the first test all the switches should be turned off and all lamps and appliances should be in position. The result of this test will show whether any switch is faulty or not. A second test should be made with the switches turned on and all the lamps and appliances removed. This will show whether the insulation resistance between the "flexibles" connecting the ceiling roses with the lamp holders, etc., is satisfactory.

The results of insulation tests give only a partial indication of the way in which the wiring of a building has been done and the quality of the materials used. If the house be damp the insulation will probably come out

low no matter how carefully the wiring has been done. If the house be dry the insulation resistances will probably come out very high even although the insulating materials used be of poor quality and the joints be made in the most careless manner.

The forty-first of the Board of Trade Regulations for securing the public from a "bad and inefficient supply of the electric light" is as follows :—

"The undertakers shall not connect the wiring and fittings on a consumer's premises with their mains unless they are reasonably satisfied that the connexion would not cause a leakage from those wires and fittings exceeding one ten thousandth part of the maximum supply current to the premises; and where the undertakers decline to make such connexion they shall serve upon the consumer a notice stating their reason for so declining."

This is usually taken to mean that, if V be the declared pressure at the consumer's terminals and F be the insulation resistance to earth of the house wiring, V/F must be less than the ten thousandth part of the maximum supply current. V/F, however, is a purely imaginary current. To make this clear we shall consider the case of a house the wiring of which is connected with two of the mains of a direct current 3-wire system of supply.

On open circuit, the potentials to earth of the house mains are the same as the potentials of the supply mains to which they are attached. As the potential difference drop on closed circuit is at the most 2 per cent., we see that no great error is made by the assumption that the potential to earth of a main is constant at all points of its length whatever may be the load. In order to simplify the theory we shall make this assumption. As the insulation resistance of the coverings of the mains is not infinite, leakage

currents will always be flowing either from the copper to the earth or vice versa. It is convenient to divide the paths of the leakage current into three groups. In a path of the first group, the current flows between the copper of one main and the earth. In a path of the second group it flows between the copper of the other main and the earth, and in a path of the third group the current flows from one main to the other without passing through the "earth." The path, for example, may be from one main to the other across the surface of a porcelain switch which may be excellently insulated from the earth. The point of this path, therefore, which is at zero potential must not be considered as belonging to the "earth."

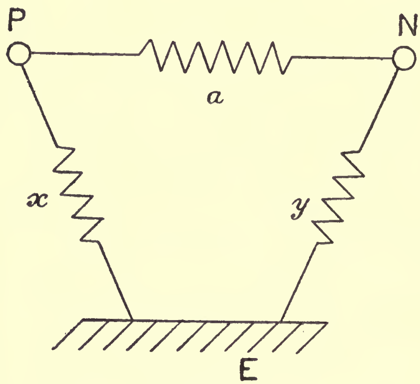

Fig. 28.

Let P and N (Fig. 28) denote the cross sections of the conductors of the house mains. Let x denote the resultant resistance of the first group of leakage paths which we suppose connects P with the earth E. Similarly let y denote the resistance of the second group connecting N with E, and a the resistance of the third group of leakage paths which are all insulated from the earth. Strictly speaking the values of x, y, and a vary with the number of switches closed and with the number of lamps which are taken from their sockets. To fix our ideas we shall suppose that the readings are taken when all the switches are on and all the lamps are removed from their sockets,

the mains being put in metallic connexion during the insulation test to earth. In this case the insulation resistance F to earth is given by

$$F = xy/(x+y),$$

and the insulation resistance R between the mains by

$$R = a(x+y)/(x+y+a).$$

The values of x, y, and a, therefore, cannot be determined from a knowledge of F and R only.

If V_1 and V_2 be the potentials of the mains P and N, the leakage current from the main P to earth will be V_1/x and the leakage current from N to earth will be V_2/y. We shall also have a leakage current $(V_1 - V_2)/a$ beween the mains. It is not clear, however, whether the greatest of these currents taken singly or the sum of the numerical values of the three is the "leakage current" specified by the Board of Trade rules.

From the point of view of the public, the considerations which limit the magnitudes of the leakage currents are the risk of fire and the damage done by electrolysis. The fire risk is the more important. From this point of view the heating effects which are measured by V_1^2/x, V_2^2/y and $(V_1 - V_2)^2/a$ respectively govern the danger. If the rules are to be equitable, the maximum permissible heating effects should be the same in all cases. If we double the voltage, therefore, the insulation resistance should be quadrupled. It is to be noticed that for given values of x, y and a the danger will be less the more distributed are the leakage paths, and the danger will be greatest when the leakage paths are concentrated at one spot. If, however, the values of x, y, and a are sufficiently high, the leakage power will be so small that there is no danger of fire even if there is only one fault. It is important, therefore, to know their values.

Measurement of the fault resistances

The best instrument to use for the measurement of the insulation resistance of the wiring of a building is a portable high voltage generator and an ohmmeter. These are combined in an instrument called the "megger" described below. The method of procedure is as follows :—

1. Measure the resistance X between P and E, when N is connected with E by a piece of wire. A water-pipe makes an excellent earth connexion. In practice it is customary to make this measurement at the main fuse block. We take out the fuses and connect one terminal of the ohmmeter to the end of the house main P where it joins the fuse block. N is connected with the water-pipe and so also is the other terminal of the ohmmeter. On turning the handle of the generator, the pointer of the ohmmeter gives the value of X directly.

2. Measure the resistance Y of N to earth when P is earthed.

3. Measure the insulation resistance F between P and N in parallel and the earth.

Our equations are,

$$1/x+1/a=1/X, \quad \ldots \quad (1),$$

$$1/y+1/a=1/Y, \quad \ldots \quad (2),$$

and

$$1/x+1/y=1/F, \quad \ldots \quad (3).$$

Hence, by addition, we find that

$$1/x+1/y+1/a=(1/X+1/Y+1/F)/2,$$

and therefore, by (2), we have

$$1/x=(1/X-1/Y+1/F)/2,$$

by (1),
$$1/y=(-1/X+1/Y+1/F)/2,$$

and by (3),
$$1/a=(1/X+1/Y-1/F)/2.$$

The reciprocals of x, y, and a are thus found in terms of measured quantities and so x, y, and a can be found. It

is to be noticed that when the sum of the reciprocals of two of the quantities X, Y, and F is nearly equal to the reciprocal of the third quantity, a small percentage error in the determination of any of them will make a large percentage error in the computed value of one of the quantities x, y, or a.

As an example let us suppose that X, Y, and F are found to be 1·98, 2·38 and 4·09 megohms respectively. In this case

$$\begin{aligned} 1/x &= (1/1{\cdot}98 - 1/2{\cdot}38 + 1/4{\cdot}09)/2 \\ &= (0{\cdot}5051 - 0{\cdot}4202 + 0{\cdot}2445)/2 \\ &= 0{\cdot}1647, \end{aligned}$$

and therefore, $x = 6{\cdot}07$ megohms. The above calculation is best made with the help of a table of the reciprocals of numbers. Similarly we find that $y = 12{\cdot}5$, and $a = 2{\cdot}94$ megohms. The fault resistance, therefore, of the main N is practically double that of the main P. Hence, unless there is any special reason to the contrary, it would be advisable to connect N with the supply main which is at the higher potential.

In practice, the resistance to earth $xy/(x+y)$, which equals 4·09 megohms, and the insulation resistance $a(x+y)/(x+y+a)$, which equals 2·54 megohms, are the quantities which are measured. But as a knowledge of these two quantities only is not sufficient to enable us to find out the values of x, y, and a, we cannot determine the leakage power or the leakage currents. We know that both x and y are separately greater than the insulation resistance to earth, and that a is greater than the insulation resistance between the mains. Hence we see that 4·09 is the minimum possible value of either x or y, and that a is not less than 2·54. Since, however, the actual values of x, y, and a can be found by the above method in a few minutes it is always advisable to find them as they give important information

about the relative values of the insulation resistance of the two mains.

In making the above test we have supposed that the readings are taken when all the switches are closed, and all the lamps and other appliances are removed from their sockets or disconnected. In this case the main part of the leakage is generally taking place across the flexible wires used in the fittings, and the value of a found by the test corresponds to the value of a when all the lights, etc., in the building are switched on. If now all the lamp switches are turned off, and all the lamps are in position, a new test can be made to see if there is any important alteration in the values of x, y, or a. The values found in this case enable us to find the leakage currents when all the switches of the consuming devices are turned off.

There is still a possible source of leakage that we have not yet considered, namely, the direct leakage between the terminals of the glow lamp itself. The terminals usually consist of pieces of brass separated from one another and from the collar of the lamp by plaster of Paris. If they are not well made, there may be considerable leakage taking place between the terminals or, if the socket for the lamp be in connexion with the earth, between the terminals and the collar. Leakage to earth through the collar of a lamp lowers the apparent fault resistance of a main. If, however, we make a test with the lamps in position, and another with the lamps removed, we can easily find out if the lamps are at fault between the leading in wires and the collar. To measure the insulation resistance of the plaster between the contact pieces is difficult as they are directly connected by the filament. It is advisable, therefore, to break the filament of a sample lamp in order to test this resistance. In good lamps it

ought to be exceedingly high, but the standard of 1,000 megohms suggested by the Engineering Standards Committee (1907) is generally considered to be excessive.

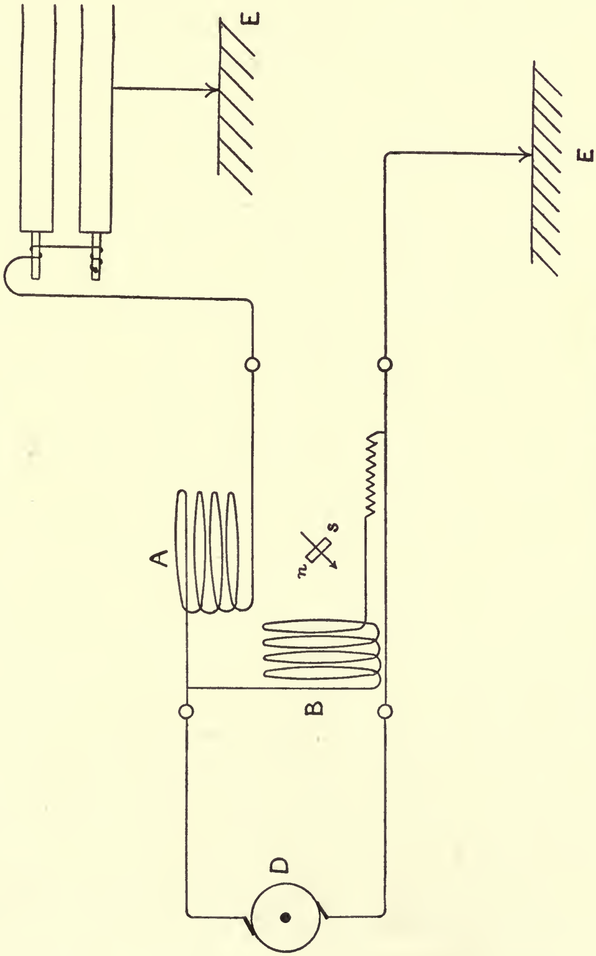

FIG. 29.—Ohmmeter and Generator.

Ohmmeter and generator

For testing the insulation resistance of the electric wiring in a building an ohmmeter and generator is usually employed. The generator

consists of a small hand dynamo D (Fig. 29) enclosed in a portable box. Instruments are made giving pressures of 100, 200, 500, or 1,000 volts. Another little box contains the ohmmeter. Two coils of wire A and B (Fig. 29) are placed with their axes making a fixed angle with one another, and a small soft iron needle ns is placed between the two. The connexions for testing the insulation resistance of the mains to earth are shown in the figure. When the handle of the generator is turned a current passes through the coil B and the resistance in series with it. If the insulation resistance of the cables to earth be infinite no current will pass through A. The needle, therefore, will set itself in the direction of the resultant magnetic force which will be parallel to the axis of the coil B. In this position the pointer will be opposite infinity on the scale of the instrument. Similarly when the insulation resistance to earth is zero, practically all the current will pass through A, and the needle will be parallel to the axis of this coil, the reading now being zero. For other values of the insulation resistance, an appreciable current passes through both coils, and the needle takes up an intermediate position. The instrument may be calibrated by putting known high resistances between its terminals and turning the handle of the generator. By means of a two-way switch, the resistance in series with B can be altered so as to increase the range of the instrument. In practical work, the readings can be trusted to within 2 or 3 per cent.

Megger

In another instrument made by Messrs. Evershed and Vignoles, called the megger, the ohmmeter and generator are combined so that they form a single instrument. The manner in which the ohmmeter principle is applied in this case is shown in Fig. 30. The ohmmeter and the generator have the same

magnetic circuit. The ohmmeter has two coils called the pressure and current coils. They are mounted on a moving axle with their axes inclined to one another. The field in the annular gap in which the current coil

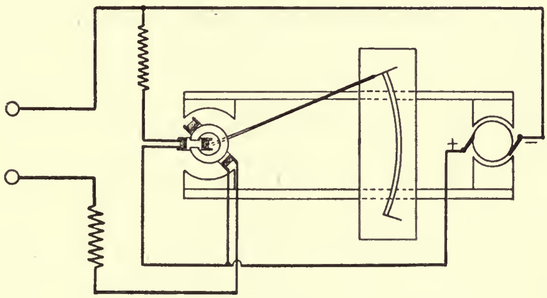

FIG. 30.—The Evershed Megger.

moves is uniform, but the pressure coil, starting from a position midway between the poles, is dragged into a field of gradually increasing strength. When there is no current in the current coil, the pressure coil is at rest with its plane midway between the poles, and the pointer reads infinity. If the resistance be zero, a large current will pass through the current coil, and the moving system will be dragged round by the forces acting on this coil into a new position of equilibrium where the pointer will read zero. For other values of the resistance and therefore of the current in the current coil, the position of equilibrium will be intermediate between these two positions and the pointer will give definite readings, and so the scale can be graduated. By suitably designing the shape of the poles so that the resistance offered by the magnetic forces acting on the pressure coil to the motion increases at a certain rate, instruments with open and evenly divided scales can be produced. The generators

are usually wound for voltages of 100, 200, 500, or 1,000. The low range instruments read from 0 to 100 megohms, and the high range instruments from 10 to 2,000 megohms.

In order to eliminate possible errors due to external fields, a differential system of winding is adopted for the pressure coil. The only thing that has to be guarded against is the demagnetisation of the magnetic circuit.

A centrifugal friction clutch is sometimes used with the generator so that, when it runs above the slipping speed, its velocity, and consequently, the E.M.F. generated is very approximately constant. When the capacity between the circuits, the insulation resistance of which is being measured, is greater than one microfarad, an appreciable condenser current will flow through the current coil if the E.M.F. make rapid periodic variations, and this current will affect the reading of the instrument. For this reason it is advisable to use a "constant pressure megger" in these cases. Both types of instrument are practically dead beat. As the total weight of the instrument is only about 18 lbs., it is extremely convenient for those tests which have to be made outside the testing room.

Electrostatic voltmeter method

Another method of measuring insulation resistance is by means of an electrostatic voltmeter and a known resistance. Let us suppose, for example, that the insulation resistance of the wiring of a building has to be measured, and let the two mains and earth be denoted by P, N, and E, respectively (Fig. 28). The procedure is as follows :—1. Disconnect the supply main connected with N from the fuse box. Let the voltmeter reading between P and N and between N and E be V_1 and V_2 respectively. Then, by Ohm's law, we have

$$y/a = V_2/V_1 \quad \dots \quad \dots \quad \dots \quad \dots \quad (1).$$

2. Disconnect the supply main connected with P and

connect the other main again to N. Let the voltmeter readings between N and P and between P and E be V_1' and V_2' respectively, then

$$x/a = V_2'/V_1' \qquad \dots \quad \dots \quad \dots \quad (2).$$

3. Finally, without altering the connexions place the resistance r between P and N, and read the voltage between the same points again. If the readings be now V_1'' and V_2'', we have

$$x/\{ar/(a+r)\} = V_2''/V_1'' \qquad \dots \quad \dots \quad (3).$$

Hence, from (2) and (3),

$$a = r(V_1'/V_2')\ (V_2''/V_1'' - V_2'/V_1').$$

The value of a is thus found and the values of x and y follow readily from (1) and (2). In connexion with this method three small carbon resistances 1, 0·1, and 0·01 of a megohm, will be found useful.

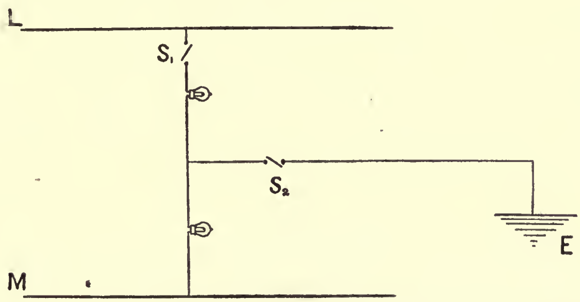

Fig. 31.—Fault indicator.

The "earth lamps" method

The following method of automatically indicating when a fault occurs on either of the mains of a 2-wire distributing system is known as the "earth lamps" method. Two 8-candle power lamps are connected in series between the mains at the distributing board. The wire joining them is connected with a water pipe by means of a switch S_2 (Fig. 31). If this switch is open and S_1 is closed both lamps will

burn dimly as the pressure between their terminals will only be half that of the supply mains. Let us now suppose that the switch S_2 is closed. If the fault resistance of each main be the same, no change in the relative brightness of the lamps will ensue, but if the fault resistance of one of them be appreciably lower than that of the other, the lamp next the faulty main will be duller than the other. On a 100 volt installation, having an insulation resistance greater than 0·1 of a megohm, the effect of earthing either of the mains through a 5,000 ohm resistance can easily be detected by the earth lamps.

It has to be carefully noticed, however, that the mere fact that opening and closing the switch S_2 has no appreciable effect on the relative brightness of the lamps is not a certain indication that there are no faults on the mains. It may only indicate that the faults are equally balanced between the two mains. If the lamp connected with M glow brightly when S_1 is open and S_2 closed this will show that the fault resistance of L is small compared with the resistance of the lamp (about 300 ohms).

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INSULATION RESISTANCE OF NETWORKS

CHAPTER VI

Insulation Resistance of Networks

Insulation resistance—Measuring fault and insulation resistance in a 2-wire system—3-wire system—Graphical construction for potentials—General theorem—Measurement of insulation resistance—Example—Regulating the potential of the mains—Leak in the positive outer—Leak in the middle main—Numerical example—Energy expended in earth currents—Leakage currents—Numerical examples—The values of f_1, f_2, and f_3—References.

Insulation resistance THE practically universal adoption of pressures of supply greater than 200 volts has brought into prominence the importance of knowing the insulation resistance of the various portions into which a network of wires, for supplying electric power, can usually be divided. The insulation resistance of a network to earth is defined to be the resistance between all the conductors of the network connected in parallel and the earth. In this chapter we shall describe methods of measuring this resistance and we shall also show how a knowledge of its value gives us important information as to the leakage currents and consequent power losses in the network. When a regular record is made of the insulation resistance not only of the whole network but also of the various portions of it, timely notice is often given, by a gradual fall in the value of the resistance, of the development of a fault. This fault can in most cases be readily located by the methods described in the next

chapter, and rectified. We shall first describe how the fault resistance of each of the mains of a 2-wire network can be found by means of a voltmeter, and an ammeter or a resistance of known value. In some cases the resistance of the voltmeter itself can be utilized as the known resistance.

Fault resistance

Let P and N (Fig. 32) denote the cross sections of the two mains, the pressure V between which

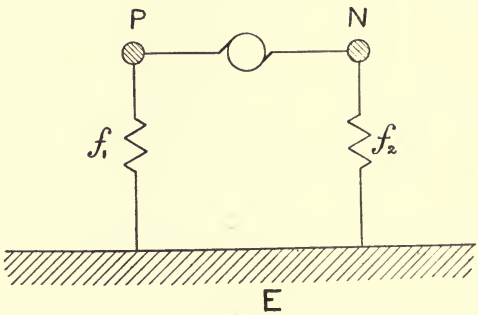

Fig. 32.

is kept constant by means of a dynamo or battery. By the fault resistance f_1 of the positive main P, we denote the combined resistance of all those stray paths from it to earth along which leakage currents flow, and similarly f_2 denotes the resultant resistance of the paths, in the insulating materials used, through which the current flows from the earth to the negative main N. We do not consider that a conductor at zero potential belongs to "earth" unless it is in good electrical connexion with earth. A metallic portion of a switch, for instance, mounted on a porcelain base may be at zero potential, and yet, the resistance of any stray paths from it to either main are not included in f_1 or f_2. Similarly any part of the path between P and N at zero potential does not belong to "earth" unless the resistance between it and "earth" —a water pipe for example—is comparable in magnitude

with the joint resistance of the direct paths to earth from P or N.

In practice we may consider that, to a first approximation, f_1 and f_2 are independent of the load between P and N. When we switch on a lamp between P and N, a portion of the connecting wire leading to the lamp is added on to the positive main. This portion previously to switching on would be at the same potential as the negative main and would be virtually part of it. If there was a leak in the portion of it beyond the lamp, we see that when the switch is open this leak is credited to the negative main but after it is closed to the positive main. In this case f_1 and f_2 will vary with the load.

If V_1 denote the potential of the positive main we shall assume that V_1/f_1 gives the value of the leakage current from this main. On open circuit, this assumption is admissible. On a heavy load, it is admissible as a rough approximation. If the voltage drop be not more than 5 per cent. and if the service circuits are well insulated, the inaccuracy introduced by our assumption will not, in the great majority of cases, be greater than 5 per cent.

The insulation resistance F of the network is given by

$$1/F = 1/f_1 + 1/f_2 \quad .. \quad .. \quad .. \quad .. \quad (1).$$

Hence, if closing a switch transfer a leaky path from f_2 to f_1, the value of F is unaltered.

Measuring fault and insulation resistance in a 2-wire system

In a 2-wire system when a voltmeter is available, the ratio of the fault resistances of the two mains can be determined immediately. We shall first suppose that the voltmeter is not electrostatic and that its resistance is R. When it is connected between P (Fig. 32) and a water pipe or other good earth let the reading be V_1'. Similarly when connected between N and earth let it read V_2'. In this case V_2' will be a negative quantity. In the first case, since by

Kirchhoff's law, the sum of the currents through R and f_1 must equal the current flowing from the earth through f_2, we have

$$V_1'/f_1+V_1'/R=(V-V_1')/f_2 \quad .. \quad (2).$$

In the second case, we have

$$-V_2'/f_2-V_2'/R=(V+V_2')/f_1 \quad .. \quad (3).$$

Hence it readily follows that

$$f_1/f_2 = - V_1'/V_2' \quad .. \quad .. \quad .. \quad .. \quad (4),$$

$$f_1 = - R\{(V-V_1'+V_2')/V_2'\} \quad .. \quad (5),$$

and

$$f_2 = R\{(V-V_1'+V_2')/V_1'\} \quad .. \quad (6).$$

From (1), (5), and (6), we see also that

$$F=R\{V/(V_1'-V_2')-1\} \quad .. \quad .. \quad (7).$$

For example, suppose that the resistance R of the voltmeter is 1,000 ohms, that $V=220$, $V_1'=160$ and $V_2'=-20$ volts, respectively, then

by (5), $f_1 = 1{,}000\{(220-160-20)/20\}=2{,}000$ ohms,
by (6), $f_2 = 1{,}000\{(220-160-20)/160\}=250$ ohms,
and by (7), $F = 1{,}000\{220/180-1\}=222$ ohms nearly.

Let us now suppose that an electrostatic voltmeter is used, and let V_1 and V_2 be the potentials of the two mains to earth respectively. In this case, the reading of the voltmeter when connected between the positive main and earth will give V_1 directly, and similarly the reading between the negative main and earth will give $-V_2$. Equations (2) and (3) may now be written.

$$V_1/f_1=(V-V_1)/f_2=V/(f_1+f_2) \quad .. \quad .. \quad (8),$$

and

$$-V_2/f_2=(V+V_2)/f_1=V/(f_1+f_2) \quad .. \quad .. \quad (9).$$

These equations show us that $f_1/f_2=V_1/(V-V_1)$, and hence, when V is known, a single reading V_1 of the voltmeter gives us the ratio of the fault resistances. In order to find their absolute values, however, further measurements must be made. For example, we may connect between the positive main P and earth a resistance and a milli-ammeter in series.

If C be the reading of the ammeter and V_1' the new reading of the electrostatic voltmeter, we have, by Kirchhoff's law

$$V_1'/f_1+C=(V-V_1')/f_2,$$

and by (8), $$V_1/f_1=(V-V_1)/f_2,$$

and thus, subtracting, $(V_1-V_1')/f_1-C=-(V_1-V_1')/f_2$.

Hence, $$F=(V_1-V_1')/C \quad \ldots \quad (10).$$

From (8) and (9), we also have,

$$f_1=(-V/V_2)F \quad \ldots \quad (11),$$

and $$f_2=(V/V_1)F \quad \ldots \quad (12).$$

As an example, let us suppose that $V=200$, $V_1=150$, and $V_1'=50$ volts, and that C equals 0·0010 of an ampere. We find, by (10), that

$$F=(150-50)/0{\cdot}001=100{,}000 \text{ ohms},$$

and by (11) and (12), that

$$f_1=(200/50)100000=400{,}000 \text{ ohms nearly},$$

and $$f_2=(200/150)100000=133{,}000 \text{ ohms nearly}.$$

By (8) and (9), we see that the power expended in the leakage currents to earth $V_1^2/f_1+V_2^2/f_2$, equals $V^2/(f_1+f_2)$. Hence any diminution in the value of f_1+f_2 always increases the power loss due to leakage currents.

Again, since

$$\begin{aligned}V_1^2/f_1+V_2^2/f_2 &= V_1^2/f_1+(V-V_1)^2/f_2\\ &=(1/F)\{V_1-(F/f_2)V\}^2+V^2/(f_1+f_2),\end{aligned}$$

we see that, if we regard V_1 as the only variable quantity, the expression for the power lost has its minimum value, when $V_1=(F/f_2)V$, that is, when Ohm's law is obeyed. We conclude therefore that if the potential difference between the mains be maintained constant, then as the fault resistances vary, the potentials of the mains vary always in such a way that the energy expended in leakage currents is a minimum (*see* Chapter I).

3-wire system

We shall now consider how the potentials of the three mains in a 3-wire system of distribution vary with the fault resistances of the three mains. As practically all direct current networks are supplied on the 3-wire system, this problem is one of considerable practical importance.

Let P, M, and N (Fig. 33) be the sections of the positive,

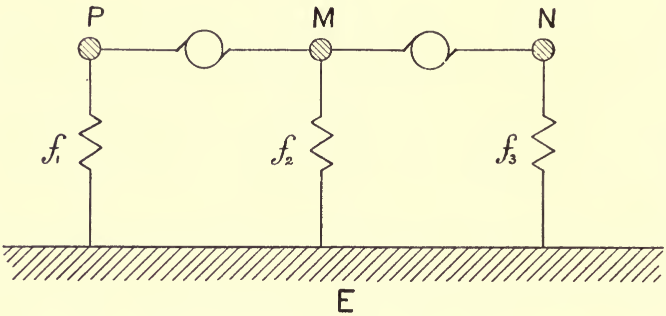

FIG. 33.

middle, and negative mains of the system and let f_1, f_2, and f_3, be the fault resistances of these mains respectively. By the fault resistance f_1 we denote the resultant resistance of all the leakage paths from the main P to earth which do not pass through the main M. If the potential of the main M be positive and there are lamps switched on between P and M, it is obvious that there will be leakage paths to earth through these lamps and then through the insulation of the main M. Even when there is no load between P and M we may have current flowing along leakage paths from P and M, and then to earth. It has to be remembered that these leakage paths directly connecting the mains and insulated from earth are not included in the fault resistances f_1, f_2, and f_3. These values merely give the resultant resistances of the *direct* leakage paths to earth from each main.

The insulation resistance F of the network is defined by the equation

$$1/F = 1/f_1 + 1/f_2 + 1/f_3.$$

It is therefore the insulation resistance between the three mains in parallel and the earth. F generally remains approximately constant at all loads, for when a switch is turned on between the positive main and the middle main, for instance, some of the leakage paths may be taken from one main and given to the other, but usually $1/f_1 + 1/f_2$ remains very approximately constant. When, however, a double pole switch is used for a leaky subcircuit, F is diminished when the switch is turned on.

In Fig. 33 let V_1, V_2, and V_3, be the potentials of the three mains P, M, and N. Since there can be no accumulation of electricity in the earth, we have, by Kirchhoff's law,

$$V_1/f_1 + V_2/f_2 + V_3/f_3 = 0 \quad .. \quad .. \quad (13).$$

We may either have V_2 and V_3 negative, or V_3 alone may be negative. At the supply station the potential differences between the mains P and M, and between M and N, are each maintained constant and equal to V (suppose). Hence

$$\left.\begin{aligned} V_1 &= V_2 + V \\ V_3 &= V_2 - V \end{aligned}\right\} \quad .. \quad .. \quad .. \quad (14).$$

Substituting for V_1 and V_3 from (14) in (13) we get a simple equation from which V_2 is easily found in terms of V, f_1, f_2, and f_3. Hence also, from (14), we find V_1 and V_3 in terms of these quantities. The following graphical construction is quite as simple as this method and is easier to apply in practice.

Graphical construction for potentials

Draw a line PN (Fig. 34) and make $PM = MN = V$. Place particles of mass $1/f_1$, $1/f_2$, and $1/f_3$, at P, M, and N, respectively and let G be their centre of gravity. We shall consider that lines measured in the direction GP are positive and in

the direction GN negative. Taking moments about G, we have

$$GP/f_1+GM/f_2+GN/f_3=0 \quad \ldots \quad \ldots \quad \ldots \quad (15).$$

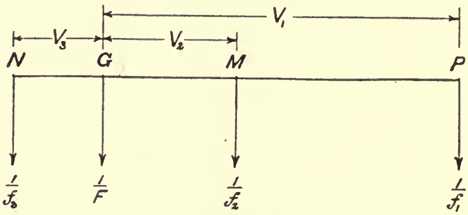

FIG. 34. Statical diagram illustrating the connexions between the potentials and the fault resistances of a three-wire distributing system.

We also have

$$\left.\begin{aligned} GP&=GM+V \\ \text{and} \qquad GN&=GM-V \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (16).$$

Comparing (15) and (16) with (13) and (14), we see at once that

$$GP=V_1,\ GM=V_2 \text{ and } GN=V_3.$$

To find the potentials of the mains, therefore, when the fault resistances f_1, f_2, and f_3, are known, we proceed as follows :—Choosing a suitable scale draw a straight line NP (Fig. 34) to represent $2V$, where V is the voltage of supply. Bisect this line in M, and find the centre of gravity G of masses $1/f_1$, $1/f_2$, and $1/f_3$, placed at P, M, and N respectively. Then the potentials of the three mains are GP, GM, and GN respectively.

General theorem

In general, if we have n mains whose fault resistances are f_1, f_2, f_3, .. and if the potential differences between them are V, V', V'', .. the potentials V_1, V_2, V_3, .. of the mains are given by the following construction. Draw a straight line P_1P_n the length of which represents $V+V'+V''+$ . . . Mark the points P_2, P_3, .. on it, where $P_1P_2=V$, $P_2P_3=V'$, etc. Place particles of mass $1/f_1$, $1/f_2$, .. at P_1, P_2, .. respec-

tively, and let G be their centre of gravity. Then it is easy to see that

$$V_1=P_1G, \ V_2=P_2G, \ \ldots$$

Measurement of insulation resistance

Let us suppose that the middle main is connected with earth at the generating station through a small resistance and an ammeter, the reading of which is C. Let us also suppose that the voltmeter connected between the middle main and earth reads V_2. If we now break the current in the earth circuit so that the ammeter reads zero, the voltmeter will assume a new value V_2', which will be numerically greater than V_2. If the voltmeter be electrostatic, the insulation resistance F of the network is given by

$$F=(V_2'-V_2)/C. \quad \ldots \quad \ldots \quad \ldots \quad (17).$$

If the voltmeter have a resistance R, we obviously have

$$FR/(F+R)=(V_2'-V_2)/C=F' \text{ (suppose)},$$

and thus,

$$F=F'R/(R-F'). \quad \ldots \quad \ldots \quad \ldots \quad (18).$$

In either case the insulation resistance is found almost at once.

We may prove formula (17) as follows. Let x denote the resistance of the earth connexion with the middle main, and let the voltmeter be electrostatic. Then, by Kirchhoff's law, we have,

$$V_1/f_1+V_2/f_2+V_2/x+V_3/f_3=0 \quad \ldots \quad \ldots \quad (19),$$

and

$$V_1'/f_1+V_2'/f_2+V_3'/f_3=0 \quad \ldots \quad \ldots \quad (20).$$

We also have

$$V_1-V_2 = V_2-V_3=V,$$

and

$$V_1'-V_2'=V_2'-V_3'=V,$$

and therefore,

$$V_1'-V_1= V_2'-V_2=V_3'-V_3 \ldots \quad (21).$$

Hence, by subtracting (19) from (20), we get

$$(V_1'-V_1)/f_1+(V_2'-V_2)/f_2+(V_3'-V_3)/f_3=V_2/x=C,$$

and therefore, by (21), $1/f_1+1/f_2+1/f_3=C/(V_2'-V_2)$,

and thus,

$$F=(V_2'-V_2)/C.$$

When the voltmeter has a resistance R, we may consider that it forms one of the leakage paths to earth on the middle main, and hence, as we have shown above, the formula can be suitably modified without difficulty.

Example

Let us suppose that initially the potential of the middle main was 8 volts, and that the reading on the ammeter was 3·5 amperes. Let us also suppose that when the earth connexion was broken the voltmeter read 112. Then, if the voltmeter is electrostatic, we get by (17),

$$F = (112-8)/3 \cdot 5 = 29 \cdot 7 \text{ ohms nearly.}$$

If the voltmeter had a resistance of 400 ohms, we find, by (18), that

$$F = 29 \cdot 7 \times 400/(400-29 \cdot 7)$$
$$= 32 \cdot 1 \text{ ohms.}$$

In practice, it is sometimes more convenient to connect the positive outer through a resistance and an ammeter to earth. The earth connexion on the middle main being opened, let V_1 be the potential of the positive outer. When the switch on the artificial leak on the positive outer is closed, let C be the reading of the ammeter and V_1' the new reading of the voltmeter. Then, proceeding as before, it is easy to show that

$$F = (V_1 - V_1')/C.$$

Regulating the potentials of the mains

The maximum pressure of supply, between "any pair of terminals," to the ordinary consumer is fixed by the Board of Trade at 250 volts. The object of this regulation is to prevent shocks, at pressures greater than 250 volts, being accidentally received. If, however, the absolute value of the potential to earth of any terminal be greater than 250 volts, it is obvious that possible shocks can be obtained between this terminal and a gas or water pipe or a damp wall or floor. To carry out the object of the regulation, therefore, it is necessary to

prevent the potential of any terminal from being permanently greater than 250 volts. We have seen above that the values of the potentials of the mains depend only on the pressure maintained between them, and on the fault resistances. The graphical construction for these potentials (Fig. 34) also shows us that by making a large artificial leak on the middle main, so that $1/f_2$ is large compared with either $1/f_1$ or $1/f_3$, we can anchor the potential of the middle main so that it never differs much from zero, and so, also, that the potentials of the positive and negative outers never differ much from $+V$ and $-V$ respectively, where V is the pressure of the supply.

It is found, in practice, that the insulation resistance of the negative outer of a 3-wire network is generally much smaller than the insulation resistances of the other mains. The flow of leakage current from the earth seems to force moisture, by a phenomenon similar to endosmosis, into the insulating covering of the main, and thus lowers its resistance. In practice, the negative outer of an insulated 3-wire network is generally at a small negative potential. For example, in a large 3-wire system in London, the potentials of the mains were generally about 190, 85, and —20 volts from earth respectively for many years. If the voltage of supply had been doubled the potential of the positive main would have been 380 and it would be clearly undesirable to have parts of lampholders and switches in damp cellars, etc., at this potential. It would therefore have been necessary to prevent the potential of the positive outer from exceeding 250 volts, and this could be done by making an artificial leak on either the positive outer or the middle main. We shall now calculate the values of the resistances of the leaks which would be necessary in order to reduce the potential of the positive outer to a given value.

Leak in the positive outer

Let us suppose that V_1, V_2, and V_3, are the potentials of the three mains, and that F is the insulation resistance of the network to earth. Let x be the resistance which has to be connected between P and earth in order to reduce its potential V_1 to the required value V_1'. If C be the current in the leak we have, by (17),

$$C = (V_1 - V_1')/F,$$

but C is also equal to V_1'/x, and thus

$$x = V_1'F/(V_1 - V_1') \quad .. \quad .. \quad .. \quad (22).$$

Leak in the middle main

If we earth the middle main through a resistance y, we have

$$V_2'/y = (V_2 - V_2')F,$$

and thus,

$$y = V_2'F/(V_2 - V_2') \quad .. \quad .. \quad (23).$$

It has to be remembered that $V_1 - V_1' = V_2 - V_2'$, and thus the current in the earth connexion, $(V_1 - V_1')/F$, can be predicted beforehand.

Numerical example

Let us suppose that initially $V_1 = 300$, $V_2 = 100$, and $V_3 = -100$ volts. Let us also suppose that F is 10 ohms. We shall calculate the value of the resistance x which has to be placed between the positive outer and earth in order to reduce its potential to 250 volts. We have, by (22),

$$x = V_1'F/(V_1 - V_1') = 250 \times 10/50 = 50 \text{ ohms}.$$

The current in this leak would be 250/50, that is, 5 amperes. In the event of a dead earth occurring in the negative outer, the potential of P would be nearly 400 volts, and the maximum value of the current in this leak would be 400/50, that is, 8 amperes.

If it were required to reduce the middle main M to zero potential, the value of x would be given by

$$x = 200 \times 10/100 = 20 \text{ ohms},$$

the ordinary value of the current in it would be 10 amperes,

and the maximum value of this current would be 20 amperes.

We shall now calculate the value of the resistance y which has to be connected between the middle main and earth in order to reduce the potential of the positive outer to 250 volts. We have, by (23)

$$y = V_2'F/(V_2 - V_2') = V_2'F/(V_1 - V_1')$$
$$= 50 \times 10/50 = 10 \text{ ohms.}$$

The current in the leak would be the same as in the preceding case, namely 5 amperes, and as the maximum possible current occurs in it when either of the outers is dead earthed, we see that the maximum possible current through y is 20 amperes.

If the resistance of y were zero, V_2' would also be zero and thus the current in y which equals $(V_2 - V_2')/F$ would be $(100-0)/10$, that is, 10 amperes.

Energy expended in earth currents

The algebraical expression for the energy expended in earth currents is

$$V_1^2/f_1 + V_2^2/f_2 + V_3^2/f_3.$$

If we use the graphical method, shown in Fig. 34, this equals

$$PG^2/f_1 + MG^2/f_2 + NG^2/f_3.$$

If we now regard the position of G as variable, by a well-known statical theorem, this expression is a minimum when G is the centre of gravity of masses $1/f_1$, $1/f_2$, and $1/f_3$, placed at P, M, and N, respectively. But we know that this is how the potentials adjust themselves in practice, and hence they adjust themselves so that the energy expended is a minimum. An analytical proof of the general theorem for an n-wire system can be given as follows. If we suppose that x is the potential of the positive outer P, the energy expended in leakage currents is

$$x^2/f_1 + (x-V)^2/f_2 + (x-V-V')^2/f_3 + \; \ldots \; \ldots \quad (24),$$

where V, V', .. are the potential differences between

P_1 and P_2, P_2 and P_3, etc. By Kirchhoff's law, we always have

$$x/f_1+(x-V)/f_2+(x-V-V')/f_3+\ \ldots\ =0\ \ldots\ \ (25).$$

But by the differential calculus this is the equation that determines the value of x which gives to the expression (24) a maximum or a minimum value. Since the second differential coefficient of (24) equals $2/F$ it is always positive and hence (24) is a minimum when x has its working value which is given by (25).

We shall now consider the value of the increase in the power lost due to earthing the middle main. We have

$$V_1{}^2/f_1+V_2{}^2/f_2+V_3{}^2/f_3=(V+V_2)^2/f_1+V_2{}^2/f_2+(V_2-V)^2/f_3$$
$$=V^2/f_1+V^2/f_3-V_2{}^2/F.$$

If we connect the middle main directly to earth, so that V_2 is zero and $V_1=-V_3=V$, the power expended in earth leakage currents is $V^2/f_1+V^2/f_2$. Hence $V_2{}^2/F$ is the value of the increase in the power loss due to earthing the middle wire. In the numerical example given above, V_2 is 100 and F is 10, and hence we see that the loss of power entailed by earthing the middle wire would be a kilowatt. In some of the older 100 volt 3-wire systems in England F is only 2 or 3 ohms, and there have been cases where it has been less. In these cases, the increase in the loss of power due to compulsory earthing of the middle wire would be appreciable.

If we earth the middle main through a resistance x, the new values V_2' and F' of V_2 and F are given by

$$V_2'/x=(V_2-V_2')/F,$$

or $$V_2'=V_2x/(F+x)\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ (26),$$

and $$F'=Fx/(F+x).$$

The increase in the leakage power, therefore, due to earthing the middle main through a resistance x, equals

$$\begin{aligned} V_2^2/F - V_2'^2/F' &= V_2^2/F - V_2^2x^2/(F+x)^2 \times \{(F+x)/Fx\} \\ &= V_2^2/F - (V_2^2/F)\{x/(F+x)\} \\ &= V_2^2/(F+x) \quad \ldots \quad (27). \end{aligned}$$

Leakage currents

It is important to notice that all methods of preventing the potentials of the mains from rising above 250 volts increase the leakage currents to earth. As these currents are always flowing it is desirable to keep them as small as possible owing to the electrolytic damage they may do. In the Board of Trade conditions for the approval of earthing, it is provided that a record of the current to earth through the earth connexion shall be kept, and that if at any time it exceeds the one-thousandth part of the maximum current of supply, immediate steps shall be taken to improve the insulation of the system. Now the current that is measured in this case is the difference between the leakage currents from the positive and negative mains. Even when the fault resistances of the three mains are very low, yet if the fault resistances of the two outers be nearly equal, the current in the earth connexion may be very small. Hence the current in the earth connexion is no sure guide as to the insulation of the network. A better rule is to insist that the insulation resistance F of the network, when the earth connexion is removed, is always above a certain value. In order to get a rough idea of a suitable minimum value for the insulation resistance of a network, we shall now consider the eighth of the Board of Trade regulations (A). This regulation is as follows :—

" 8. *Maintenance of Insulation.*—The insulation of every complete circuit used for the supply of energy, including all machinery, apparatus and devices forming part of, or in connexion with, such circuit, shall be so maintained that the leakage current shall not under any conditions exceed one-thousandth part of the maximum supply current ; and

suitable means shall be provided for the immediate indication and localization of leakage. Every leakage shall be remedied without delay.

"Every such circuit shall be tested for insulation at least once in every week, and the undertakers shall duly record the results of the testings.

"Provided that where the Board of Trade have approved of any part of any electric circuit being connected with earth, the provisions of this regulation shall not apply to that circuit so long as the connexion with earth exists."

For a 2-wire system this rule is clear. It proceeds on the assumption that the permissible leakage power must always be the same fraction of the total output. In other words, if we double the pressure of supply, the output remaining the same, the lowest permissible value of the insulation resistance is increased four times. Hence, from the fire risk point of view, the various 2-wire systems of supply are treated equitably. The systems, however, that use lower pressures are allowed to have larger leakage currents, and in the course of time the electrolytic damage done by them may be appreciable.

When we try to apply the above rule to 3-wire networks we are met with the difficulty that there is no simple method of finding f_1, f_2, and f_3, and hence the determination of the earth leakage currents is difficult. We can, however, easily determine the insulation resistance F to earth of the network and the potentials V_1, V_2, and V_3, of the mains. The question therefore arises, having given the potentials of the mains and the insulation resistance of the network, can we determine superior limits to the value of the greatest leakage current from a main and to the value of the leakage power expended in the earth currents from the mains. If we know the maximum and minimum possible values

of these quantities for given values of F, V_1, V_2, and V_3, we may possibly be able to fix on a minimum permissible value to F in any given case. It would of course be preferable to determine f_1, f_2, and f_3, in every case, but the method of doing this described below is difficult and small percentage errors in the readings of the instruments used may lead to large percentage errors in the values of the quantities found.

Let us now suppose that the values of V, V_1, V_2, V_3, and F, are known and investigate the maximum value of the leakage current $-V_3/f_3$ to the negative main. When the middle main is at positive potential, which is the usual case in practice, the leakage current to the negative main being equal to the sum of the leakage currents from the positive and middle main will be numerically the greatest of them, and hence this is the current to which we have to find the superior limit. This is done easily from the graphical construction given in Fig. 34. In the problem considered, G is fixed, and so also is the sum $1/F$ of the masses placed at P, M and N. Now $-V_3/f_3$, the leakage current to the negative main, is the moment about G of the mass $1/f_3$ placed at N. We have to find, therefore, the maximum value of this moment subject to the condition that the sum of the three masses at P, M and N is a constant quantity and that their centre of gravity G is a fixed point. If the mass at M be not zero, we can increase the values of $1/f_1$ and $1/f_3$, by dividing this mass into two portions in the ratio of $-V_3$ to V_1, and placing these portions at P and N respectively. This procedure would increase the mass at N without altering the position of G or the value of F. Hence $1/f_3$ has its maximum possible value, subject to the given conditions, when $1/f_2$ is zero. In this case,

$$1/f_1+1/f_3=1/F, \text{ and } C_{max.}=V_1/f_1=-V_3/f_3.$$

Hence $C_{max.}/V_1 - C_{max.}/V_3 = 1/f_1 + 1/f_3 = 1/F$,

and thus $C_{max.} = \{V_1 V_3/(V_3 - V_1)\}/F$

$$= \{(V - V_2)(V + V_2)/2V\}/F \quad \ldots \quad \ldots \quad (a).$$

This is also the maximum possible value of the earth leakage current from the positive outer, and it is greater than any possible current from the middle main. Hence (*a*) fixes a superior limit to the value of the earth leakage currents.

When the potentials of the mains and the insulation resistance of the network are known, proceeding as above, it is easy to show that the smallest possible value of the greatest earth current from any of the three mains occurs when there is no leakage from one of the outers. Assuming that f_1 is infinite we find that

$$C_{min.} = V_2/f_2 = -V_3/f_3,$$

and hence, proceeding as before, we get

$$C_{min.} = \{(V - V_2)/F\}(V_2/V) \ldots \quad \ldots \quad \ldots \quad (b).$$

If P denote the power expended on leakage currents, we have

$$P = V_1^2/f_1 + V_2^2/f_2 + V_3^2/f_3$$
$$= (V^2 - V_2^2)/F - V^2/f_2.$$

It is therefore a maximum when f_2 is infinite, and so

$$P_{max.} = (V^2 - V_2^2)/F = 2VC_{max.} \quad \ldots \quad \ldots \quad \ldots \quad (c).$$

The minimum value of P occurs when $1/f_2$ has its greatest value, that is, when f_1 is infinite. In this case the leakage current between the middle main and the negative outer is $C_{min.}$, and therefore

$$P_{min.} = V_2(V - V_2)/F = VC_{min.} \ldots \quad \ldots \quad \ldots \quad (d).$$

Let us now suppose that the middle wire of this system is connected with earth through a resistance of x ohms. If V_2' be the new potential of the middle wire, we have, by (23),

$$V_2'/x = (V_2 - V_2')/F.$$

As before we see that the leakage current to the negative

or from the positive outer will have its maximum value when f_2 is infinite. In this case if $C'_{max.}$ denote the maximum value of the current, and we suppose V_2' positive, we have

$$C'_{max.} = V_1'/f_1 + V_2'/x.$$

Since $1/f_1 + 1/f_3 = 1/F$, and $V_1/f_1 = -V_3/f_3$, we get

$$\begin{aligned} C'_{max.} &= V_1'(V - V_2)/2VF + (V_2 - V_2')/F \\ &= (V - V_2')(V + V_2)/2VF \quad \ldots \quad \ldots \quad \ldots \quad (a'), \end{aligned}$$

and, by (26), V_2' is given by

$$V_2' = V_2 x/(F + x).$$

Thus, since V_2' is always less than V_2, $C'_{max.}$ is always greater than $C_{max.}$ Earthing the middle main through a resistance, therefore, always increases the superior limit of the possible values of the leakage currents.

Similarly the minimum value $C'_{min.}$ of the leakage current occurs when f_1 is infinite, and hence

$$C'_{min.} = (V - V_2')V_2/VF \quad \ldots \quad \ldots \quad \ldots \quad (b').$$

Comparing this with (b) we see that $C'_{min.}$ is greater than $C_{min.}$.

If $P'_{max.}$ denote the maximum possible value of the power expended on leakage currents, it is not difficult to show that

$$P'_{max.} = V^2/F - \{x/(F + x)\}(V_2^2/F) \quad \ldots \quad \ldots \quad (c').$$

Similarly we can show that

$$P'_{min.} = VV_2/F - \{x/(F + x)\}(V_2^2/F) \quad \ldots \quad \ldots \quad (d').$$

It follows by comparing the corresponding formulae that $P'_{max.}$ and $P'_{min.}$ are respectively greater than $P_{max.}$ and $P_{min.}$.

In the particular case, when the middle main is dead earthed so that both x and V_2' are zero, the formulae become

$$\begin{aligned} C''_{max.} &= (V + V_2)/2F \quad \ldots \quad \ldots \quad \ldots \quad (a''), \\ C''_{min.} &= V_2/F \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (b''), \\ P''_{max.} &= V^2/F \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (c''), \\ \text{and } P''_{min.} &= VV_2/F \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (d''). \end{aligned}$$

The above formulae show that whether we earth the middle wire or not, $(V+V_2)/2F$, and *à fortiori* V/F, is a superior limit to the value of the earth current to or from any main. They also show that V^2/F is a superior limit to the power expended in earth currents.

For instance, if $V=220$ volts, and $F=25$ ohms, the power expended in leakage currents cannot be greater than $220^2/25$, that is, 1·936 kilowatts, and no earth current can be greater than V/F, that is, 8·8 amperes. If the values of V_2 and x be known, we can in general reduce these values considerably. The following numerical examples illustrate how readily the above formulae, which are due to the author, can be applied in practice.

Numerical examples Let us suppose that the maximum output of a 3-wire direct current station with 400 volts between the outers is 3,000 kws. We shall calculate the lowest insulation resistance which will ensure that no earth leakage current is greater than the thousandth part of the maximum supply current, the potential of the middle main being 40 volts.

In this case $V=200$, $V_2=40$, the maximum current of supply is (3,000,000/400), that is 7,500 amperes, and therefore the maximum leakage current must not exceed 7·5 amperes. Substituting these values in (*a*), we get

$$7{\cdot}5=\{(200-40)/F\}\{(200+40)/400\},$$

and therefore $F=12{\cdot}8$ ohms. Hence if the insulation resistance of the network be greater than 12·8 ohms the maximum value of the leakage current from any part of the three mains will be less than the thousandth part of the maximum supply current.

The maximum possible value of the power expended in the currents to earth in this case is, by (*c*),

$$P_{max.}=(200^2-40^2)/12{\cdot}8=3 \text{ kws.}$$

If the middle wire had to be dead earthed we see, by (a''), that

$$F = 16 \text{ ohms},$$

and by (c''),

$$P_{max.} = 200^2/12{\cdot}8 = 3{\cdot}125 \text{ kws.}$$

Let us now suppose that the pressure between the outer mains was reduced to 200 volts, so that $V=100$, and $V_2=20$. If the output of the station remained the same the maximum permissible leakage current would be 15 amperes. In this case for the insulated network, by (a), F must not be less than $\{(100-20)/15\}\{(100+20)/200\}$, that is, 3·2 ohms, and for the earthed network, F must not be less than 4 ohms. The values of the leakage power expended in the leakage paths would be as before 3 and 3·125 kws.

In practice, it is not permissible to have a voltage drop in the mains greater than 4 per cent., and hence (Chapter V) the maximum load on a low pressure network varies as the square of the voltage. The maximum permissible load, therefore, when the pressure is halved is only one-quarter of its original value, and thus the maximum permissible leakage current is only 3·75 amperes, and the value of F, therefore, is 16 ohms, the same value as before.

As a further example, let us suppose that the potentials of the mains of a 3-wire direct current system are 300, 100, and —100 volts, respectively. Let us also suppose that the insulation resistance F of the system is found to be 10 ohms. We shall find the limits between which the greatest of the earth currents must lie and also the limits between which the leakage power must lie, both when the network is insulated and when the middle wire is connected with earth through a resistance of 2 ohms.

By formulae (a), (b), (c), and (d), we at once find that

$$C_{max.}=(200-100)(200+100)/400\times 10=7\cdot 5 \text{ amperes},$$
$$C_{min.}=(200-100)100/200\times 10 \qquad =5 \text{ amperes},$$
$$P_{max.}=400\times 7\cdot 5 \qquad =3 \text{ kws.},$$
and $P_{min.}=200\times 5 \qquad =1$ kw.

When the resistance x of the earth connexion of the middle main is 2 ohms, we have, by (26),

$$V_2'=\{x/(F+x)\}V_2=100/6=16\cdot 7 \text{ volts approx.},$$

and therefore, $V_1'=216\cdot 7$ and $V_3'=-183\cdot 3$ volts.

We easily find by (a'), (b'), (c'), and (d'), that

$$C'_{max.}=(200-16\cdot 7)300/400\times 10 \qquad =13\cdot 75 \text{ amperes},$$
$$C'_{min.}=(200-16\cdot 7)100/200\times 10 \qquad =9\cdot 17 \text{ amperes},$$
$$P'_{max.}=200^2/10-(1/6)100^2/10 \qquad =3\cdot 83 \text{ kws.},$$
and $P'_{min.}=200\times 100/10-(1/6)100^2/10=1\cdot 83$ kws.

In this case, the current in the earth connexion is $16\cdot 7/2$, that is $8\cdot 35$ amperes.

If finally we suppose that the middle main is dead earthed so that x is zero, we have by (a''), (b''), (c''), and (d''),

$$C''_{max.}=(200+100)/2\times 10=15 \text{ amperes},$$
$$C''_{min.}=100/10 \qquad =10 \text{ amperes},$$
$$P''_{max.}=200^2/10 \qquad =4 \text{ kws.},$$
and, $P''_{min.}=200\times 100/10 \qquad =2$ kws.

We could have predicted at once that the maximum leakage current would in any case have been less than V/F, that is, 20 amperes and that the maximum leakage power could not have been greater than V^2/F, that is, 4 kilowatts. The more complicated formulae, however, give us valuable additional information.

As a final example, we shall take the values obtained by measurements made in 1900 on a large supply network in London. In this case

$$V_1=190,\ V_2=85,\ V_3=-20, \text{ and } F=2\cdot 5.$$

We shall find the limits between which the maximum value of the earth current to any of the mains must lie,

and also the limits for the leakage power. By formulae (*a*) and (*b*), we have,

$C_{max.} = (105-85)(105+85)/210 \times 2{\cdot}5 = 7{\cdot}24$ amperes,

and $C_{min.} = 85 \times 20/105 \times 2{\cdot}5 \qquad = 6{\cdot}48$ amperes.

Whatever may have been the actual values of the fault resistances of the mains, the value of the leakage current to the negative main cannot have been less than 6·48 amperes or greater than 7·24 amperes. Similarly, by (*c*) and (*d*), we find that the value of the leakage power cannot have been less than 0·68 kw. or greater than 1·52 kws.

If the middle main of this network had been earthed the current to the negative main would have had some value between 34 and 38 amperes, and the power expended in leakage currents would have had a value between 3·57 and 4·41 kws. The current in the earth connexion also would have been 34 amperes. In this case, the only advantage gained by earthing the middle wire would have been the reduction of the potential of the positive main from 190 to 105 volts. On the other hand, the leakage current to the negative would now have been doing five times the amount of electrolytic damage, and in order to maintain the balance of the potentials about 3 kws. would have to be expended in the leakage paths all the year round.

These examples illustrate that a knowledge of V_2 and F gives us most useful information about the leakage currents and the leakage power in a 3-wire network.

The values of f_1, f_2, and f_3

There is no good practical method of determining the values of f_1, f_2, and f_3. When it is permissible to arrange that, during the brief time required to take the necessary readings, the voltage between the positive and the middle may be 10 or preferably 20 per cent. different from the voltage between the negative and

the middle, the following method will give approximate values of the three fault resistances.

Measure in the ordinary way, first of all, F, V_1, V_2 and V_3. This gives us the two equations,

$$1/f_1+1/f_2+1/f_3 \quad =1/F \qquad .. \quad .. \quad (1),$$

and

$$V_1/f_1+V_2/f_2+V_3/f_3=0 \quad .. \quad .. \quad .. \quad (2).$$

Next upset the balance of the pressures so that the voltages between the two sides of the 3-wire supply are appreciably different, and measure the new values V_1', V_2', and V_3', of the potentials of the mains.

This gives us the further equation—

$$V_1'/f_1+V_2'/f_2+V_3'/f_3=0 \quad .. \quad .. \quad .. \quad (3).$$

Hence we have three equations to determine three unknown quantities $1/f_1$, $1/f_2$, and $1/f_3$, and thus solving by determinants, or otherwise, we get

$$1/f_1=(V_2V_3'—V_3V_2')/F\Delta,$$

$$1/f_2=(V_3V_1'—V_1V_3')/F\Delta,$$

and $1/f_3=(V_1V_2'—V_2V_1')/F\Delta,$

where $\Delta= V_2V_3'—V_3V_2'+V_3V_1'—V_1V_3'+V_1V_2'—V_2V_1'.$

The solution shows that a small percentage error in the value of a voltmeter reading may sometimes introduce a large error in the value of the fault resistances deduced by the formulae. Suppose, for instance, that $V_1V_2'—V_2V_1'$ is 12 and that V_1V_2' is 2,000, then a 1 per cent. error in the reading of either V_1 or V_2' could give to f_3 an impossible negative value. Hence this method has to be used with caution.

It is to be noticed that it is necessary to upset the balance of the voltages in order to get equation (3). If we merely make an artificial leak in one of the mains we get

$$V_1''/f_1+V_2''/f_2+V_3''/f_3=C \quad .. \quad .. \quad .. \quad (4),$$

where C is the current in the leak. But since $V_1—V_1''= V_2—V_2''=V_3—V_3''$, and $F=(V_1—V_1'')/C$, equation (4) can

be deduced from (1) and (2) and is therefore not an independent equation.

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FAULTS IN NETWORKS

CHAPTER VII

Faults in Networks

Faults in networks—House wiring—Earths—Short circuits—Breaks—Distributing networks—The localization of faults—Detecting faulty mains by flashing—General methods—The Hopkinson 3-ammeter method—The final methods of localization—The fall of potential method—Loop test—2-ammeter method—Induction method—Blavier's test—Example—References.

Faults in networks

THE faults that most commonly arise in practice are due to causes which can be roughly classified under three headings, short-circuits, earths, and breaks. A short circuit, or as it is generally called in America, a "cross," occurs when two conductors of opposite polarity get connected by a path of very small resistance. The consequent dangers, of fire and of the dynamos being overloaded, arising from this type of fault, are obviated in practice by means of fuses or automatic cut-outs. An earth, or a "ground" as it is sometimes called, occurs when any conductor of the network makes contact through a path of small resistance with the earth. Water pipes, for instance, make effective contact with the earth, and if a metal conductor touch a water pipe in such a way that the contact resistance is very small it makes what is called a "dead earth." Sometimes, however, the resistance of the fault is appreciable and we get what is called a partial earth. If systematic tests of the insulation resistance to earth of the wiring be not made

periodically, this kind of fault may remain undetected for a long time, and in the event of a fault developing on a main of opposite polarity there will be a risk of fire. The third kind of fault occurs when there is a partial or a total break in a conductor. It may arise owing to a terminal screw working loose, and the end of the conductor ceasing to make contact, or it may be due to an actual break in the conductor itself.

The localization of faults in a distributing network is an operation demanding not only considerable skill, but also a thorough knowledge of how the cables and feeders are arranged in the network. A detailed plan of the wiring is therefore almost essential, and ought always to be readily available. Having access to this plan, it is, as a rule, not difficult to devise a method of procedure which must ultimately locate the fault. It is always best to make the search in a methodical and thorough manner. The youthful engineer, for instance, sometimes neglects to test part of a section simply because it is easier to disconnect and make rough tests at the sectional pillars than at the underground manholes. Hence a partial earth, which could easily be detected at a manhole, may be left undetected for months.

If the search be made methodically the fault or faults cannot fail to be discovered. Sometimes the first test indicates the position of the only fault, and sometimes the faulty section is only found after having isolated and tested all the others.

House wiring

As an introduction to the more difficult case of a distributing network let us consider the method of testing for faults in a house wiring circuit. To simplify the problem, we shall consider the case of a house installed on the 2-wire system.

Earths

Let us first suppose that the insulation resistance to earth of the wiring shown in the diagram (Fig. 35) has been found to be below the standard. We have therefore to locate the section on which the partial or dead earth is situated. In Fig. 35, *CMF* represents the company's main fuse, *M* the meter, *MS* the main switch, *MF* the main fuse, and *MDB* the main distributing board. The distributing boards for the various floors are marked *DB*.

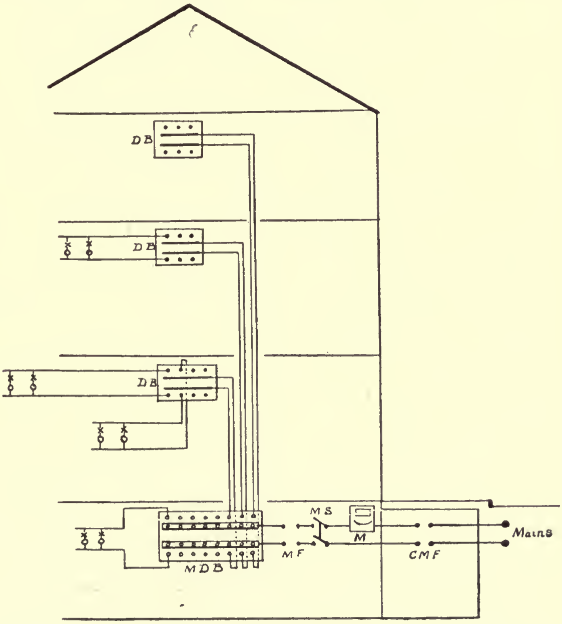

FIG. 35.—Diagram of House Wiring.

We shall suppose that the insulation test (*see* p. 100) to earth has been made at the company's main fuse *CMF*, all the lamps being in their sockets and all the switches being closed. We first open the double pole switch *MS*, and repeat the test. If the testing instrument, ohmmeter let

us suppose, now read infinity we see that the fault is not in connexion with the meter or the main switch. If, however, it reads practically the same as when the switch was closed, the fault is in the meter or the base of the switch. In the former case, a bare wire is probably making contact with the meter cover, and in the latter, the base of a switch may be a conductor. The slate, for instance, of which it is made may have metallic veins. This can generally be remedied by bushing the fixing screws with ebonite.

We have next to test the mains on the house side of the main switch *MS*. Turn off the switches on the main distributing board, one by one, and take the reading of the ohmmeter between each operation. Let us suppose, for example, that when the fourth switch is turned off the reading changes very appreciably. In this case, the fourth switch obviously controls a faulty section. By disconnecting the leads from the distributing board on No. 4 circuit, we can readily test at the main switchboard whether the fault be in the mains connecting the two boards. If these mains be free from faults we next proceed to No. 4 distributing board, and test it in the same way as the main switchboard. We thus finally locate the faulty lamp circuit. The fault may be due to a defective switch having been placed on damp plaster or an abrasion of the covering of one of the wires or flexibles may provide a path of small resistance to a neighbouring gaspipe or a steel girder. Faults are often found also in ceiling roses or lamp brackets.

Suppose that when all the switches are turned off on the main switchboard the insulation resistance measured from *MS* still reads very low. In this case we first remove the fuses in *MF*, and test the insulation resistance of the section between *MS* and *MF*, and of the base of

MF. If these resistances were satisfactory the fault or faults must be on the main switchboard.

To test the main switchboard we disconnect all the mains from it. We then join all the metallic parts on the face of the board with binding wire, and measure the insulation resistance between this wire and earth. By disconnecting the binding wire from each of the metallic portions in turn, and reading the ohmmeter between each operation, we test each portion, and thus readily locate the fault or faults. Slate of inferior quality, badly insulated from the fixing screws by defective bushing, may easily develop bad earth faults.

Short circuits

It will be seen that to locate an earth fault rapidly, more especially when its resistance is high, an ohmmeter or some other suitable testing instrument is necessary. The location of a short circuit, however, requires no instrument and is usually exceedingly simple. The blowing of the fuse generally locates the faulty circuit. We have then to examine the lamp, holder, ceiling rose, and flexible cord to find out where contact between conductors of opposite polarity is taking place.

In ordinary installations, short circuits can occur in the lamp holders, and in the flexible wires used to support the lamps. In these cases the fuse protecting the circuit generally blows at once, and thus they are not dangerous. When, however, a flexible wiring system is used, or when a switch is connected with flexible wires, a more dangerous partial short circuit can occur. Let us suppose, for example, that the switch S (Fig. 36) controls the lamps L. If a short circuit occurs at A or C, the fuse will immediately blow, but if the short circuit occurs at B, between the wires connected with the switch, the lamps will still be in circuit. Hence, although an arc will start at B, the fuses

will not blow. In this case the arc will probably move slowly up the flexible until the mains are involved, when the fuse will almost certainly blow. The risk of fire will

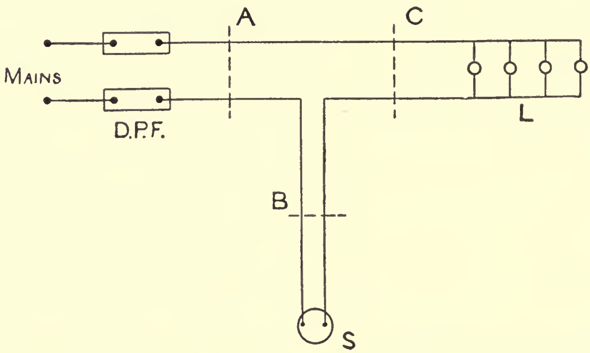

FIG. 36.—A Short Circuit at *B* is dangerous in flexible Wiring Systems.

therefore be much greater when the short occurs at *B*, than when it occurs at either *A* or *C*. To obviate this risk a safety device, due to Coninx, is sometimes employed. A third wire (Fig. 37) connected with the opposite main is

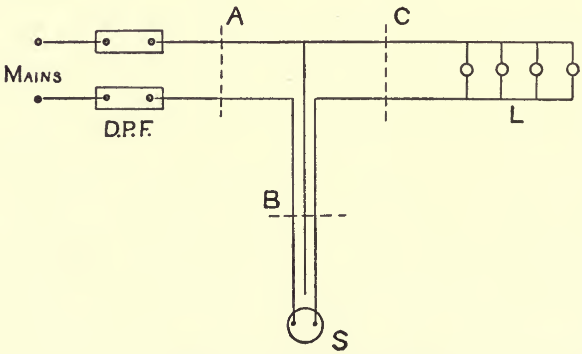

FIG. 37.—The Safety Wire.

twisted with the flexible required for the switch connexion *S*. Hence in the event of an arc occurring on a switch flexible, a dead short will be sure to occur very quickly

between the mains. This will blow the fuses before the arc has time to set fire to neighbouring objects, and so the fire risk will be minimized.

Breaks

A break in the continuity of the conductors is generally easily located when a portable voltmeter is available. If the switch be turned on we can find whether two parts of a conductor are of opposite polarity by noticing the reading of the voltmeter when its terminals are connected by means of suitable flexible conductors across the two parts. If no pressure be indicated, they are both on the same side of the break, but if the full pressure be indicated they are on opposite sides of the break. By pushing needles through the insulation, contact can be made with the conductor and the exact position of the break can often in this way be rapidly located.

Distributing networks

Methods of rapidly finding the position of faults on a distributing network are of considerable importance to the station engineer. In ordinary low-tension networks, the location is only difficult, when the network is closely netted by numerous feeders. Attention is often directed to the fault at once by the complaints of consumers, and blown fuses in the manhole section boxes or the section pillars indicate the faulty section.

Let us consider the case of a 3-wire low-tension network with its neutral earthed through a resistance of 2 ohms. In order that faults may be detected as soon as possible it is essential that daily tests of the insulation resistance of the whole system to earth be made. The chart of the recording ammeter in the earth connexion should also be closely studied to see if there is any periodic variation. If there is, it is probably due to a fault in a private installation periodically brought in and cut out of the network

by the consumer's switch. A continuous rapid oscillation of the ammeter pointer indicates that there is a defective motor armature in the circuit. If the ammeter in the earth connexion be polarized so that it indicates the direction as well as the magnitude of the current, we can tell at once which of the outers has the greater fault resistance. If the current be flowing from the middle conductor to earth the negative outer has the lower fault resistance, but if it flow in the other direction the positive outer has the lower fault resistance.

When the reading of the recording ammeter in the earth connexion is very small, it has to be particularly noticed that the insulation resistance is not necessarily high. If the fault resistance of the middle main be very low or if the fault resistances of the two outers be nearly balanced, no matter how low they may be, the reading of the recording earth ammeter may be very small. A small reading of this instrument gives no certain indication of the magnitude of the earth faults on the system, but a large reading indicates that the insulation resistance of at least one of the mains is very low.

The daily test of the insulation resistance of the network (*see* Chapter VI) gives much more information about the magnitude of the faults than the readings of the earth ammeter. A sudden fall in the value of this quantity indicates that at least one fault has suddenly developed in the network. An inspection of the chart of the recording ammeter *may* indicate the exact time at which this fault developed, and the direction of the flow of current in the earth connexion in the case of serious faults usually indicates the main in which the fault has taken place. If the earth ammeter does not indicate the direction of the current it can be readily found by Ampère's rule with the

help of a small compass. It is advisable to leave this small compass permanently in position so that the direction can always be ascertained by a glance. The polarity of the middle main with reference to the earth can also be easily found by pole testing paper.

Having found the outer on which the fault exists (the negative suppose), we increase the resistance in the earth connexion, and momentarily close an artificial leak of small resistance in the sound outer. If the fault exist on a

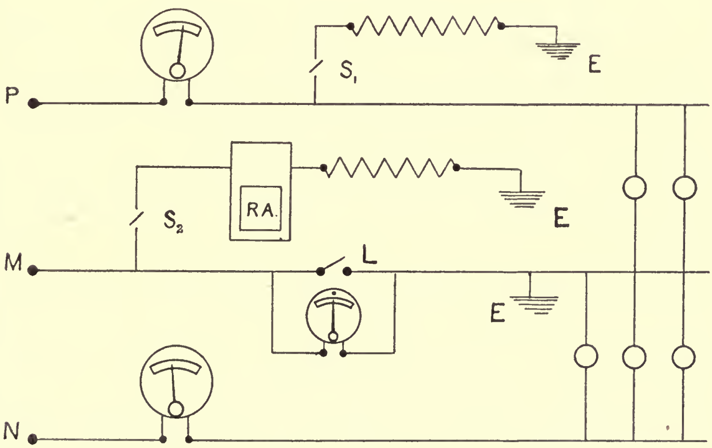

Fig. 38.—Earth on the Middle Wire.

consumer's circuit, the fuse in series with it will blow, and his complaints will determine the position of the fault. If closing momentarily the artificial leak, that is, if flashing does not clear the fault, it must lie on a part of the main protected by a large fuse, or there must also be a large fault on the middle main.

Detecting faulty mains by flashing

Let us suppose that a central zero ammeter (Fig. 38) or a current direction indicating ammeter can, by opening the switch L or by taking

out a plug, be readily put in series with the middle main. Choose some time of the day when the load is very small, and put the ammeter in circuit by opening L. Increase also the resistance of the earth connexion S_2E, or preferably, if permissible, open the switch S_2 of this connexion. Now flash the positive main P, by closing the switch S_1, and notice whether this operation produces a throw of the ammeter pointer. If it does there must be a fault on the middle main. In large networks there may always be a slight deflection of the ammeter pointer when either P or N is flashed, owing to the great length, and consequent small fault resistance of the middle main. The engineer-in-charge knows approximately the magnitudes of the throws to be expected and so an increase in the value of the throw indicates that a fault has developed. The absence of a throw also might indicate that a fault had been cleared.

When the above operation indicates that there is a fault on the middle main, we have to determine the portion of the main or feeder on which the fault is situated. If we put an ammeter in circuit with each of the neutral feeders in turn, and notice the effect in each case on the ammeter pointer of flashing the outer, a faulty feeder will be indicated by the large throw obtained when the testing ammeter is in circuit with it. If all the feeder mains are sound, the fault is on the middle distributing main, and by putting an ammeter in circuit at various points of the middle main in turn, the first point at which no abnormal throw is observed on flashing the outer is the end of the section of the middle main on which the fault is situated.

By noticing the readings of the ammeters in circuit with the feeder mains of the negative outer, when the positive outer is flashed, a faulty negative feeder can often be de-

tected. Similarly a fault on a positive feeder is indicated by a throw on its ammeter when the negative is flashed.

Let us now suppose that the existence of a fault on one of the negative feeders has been discovered. The various distributors branching from this feeder should then be transferred, one at a time, to another feeder. This may be easily done at the section pillars or at the manholes. The flashing test is repeated after each transfer, and thus, the faulty distributor is found when the transfer stops the

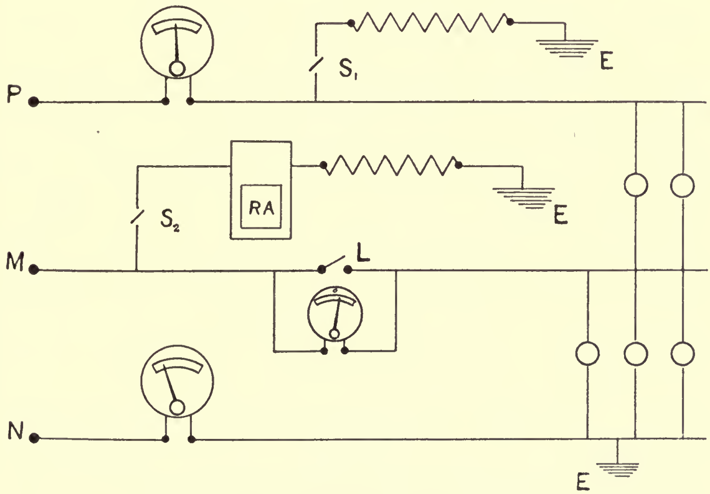

Fig. 39.—Earth on the Negative Outer.

throw on the feeder ammeter due to flashing. If the fault be small, it may be necessary to put a large resistance in the earth connexion of the middle main during the test. This test is sometimes laborious owing to the frequent journeys to the station and back between each disconnexion. It is to be noticed that none of the tests described hitherto interfere with the supply to the consumers.

When it is permissible to open the earth connexion of

the middle main, and to disconnect various sections of the network in turn, we may proceed as follows. Let us suppose, for instance, that there is an earth (Fig. 39) on the negative main. In this case there will be an appreciable reading on the recording ammeter *RA*. When we open the switch S_2, if the fault be a bad one, the potential of the positive outer from earth will be practically double of the pressure of the supply, and the potential of the middle main will be practically the same as the supply pressure. In this case a lamp will burn brightly when connected between the middle main and earth. In general, if there is an appreciable fault on the negative main, a lamp connected between the middle main and earth will glow more or less brightly. If then we connect, at the nearest network box, a lamp between the middle main and the earth, the appearance of the filament will indicate the value of the voltage V_2. The various service lines are disconnected in turn. If the faulty service line is connected with this box, there will be a sudden change in the brightness of the filament when it is disconnected. If not, we have to proceed to the various network boxes in turn and repeat the test, until the faulty service line is discovered. When a suitable portable voltmeter is available it is better to use it instead of a lamp.

When the fault is in the middle main there will be practically no current indicated by the ammeter *RA* in the earth connexion. When disconnexion of the service lines is permissible we may proceed as follows. Open the switch S_2 (Fig. 39) and make a small artificial leak in the negative outer. We then disconnect the service lines as in the last section, and proceed as before, the only difference being that the lamp will now glow when the faulty section is disconnected.

It will be seen that these methods are very simple and easy to apply. A drawback to their use is the necessity of breaking for a few seconds the continuity of the supply to individual consumers in the sections under test.

General methods The following methods of locating faults in distributing networks, described by F. Fernie, will be found trustworthy and expeditious. In modern networks, the different feeder sections are linked by fuse

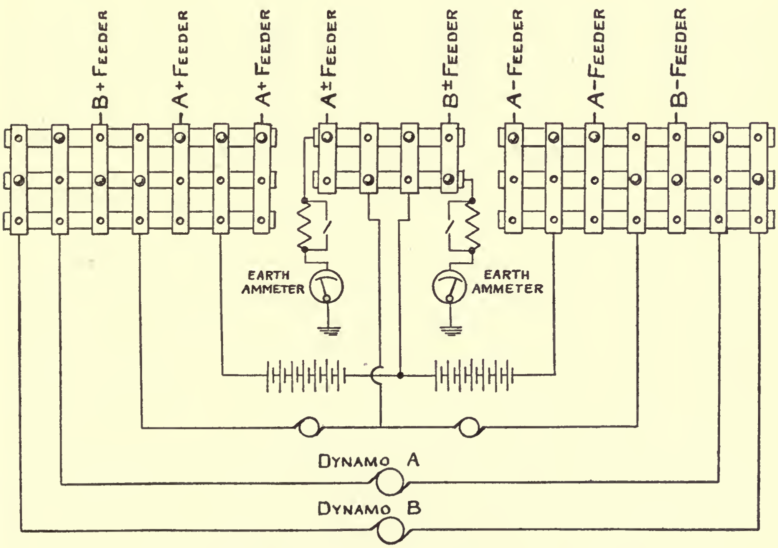

Fig. 40.—Arrangement of Switchboards during the test.

switches in pillars above ground which are opened by a key. By sending a man round on a bicycle, therefore, it is a simple matter to divide the network (Fig. 40) into two distinct sections *A* and *B*, by removing the requisite positive, negative and neutral fuses in the various pillar boxes. The neutral fuse is usually a stout piece of copper wire. If there is only one neutral bus bar at the station, a second must be extemporized. The *A* section is fed by one set

of dynamos and the B section by another set. The balancing of the A section may be done by the storage battery, and the balancing of the B section by the balancer. The neutral bus bars belonging to the A and B sections respectively are earthed through separate ammeters. If there is a fault on one of the outers the earth ammeter of the section on which the fault is situated will give a large reading. Groups of feeders on this section together with the machine to which they are connected are now transferred methodically to the other. The earth ammeters are inspected between each operation. When the faulty group is transferred, the reading on one of the ammeters will suddenly drop and the reading on the other suddenly rise. The fault is thus localized to this group.

The feeders of the group on which the fault is situated are now transferred back to the other section, one by one, until the faulty feeder is discovered. This faulty feeder may then be subdivided further, and the fault be localized to a few streets. By disconnecting at the service boxes, it can then be determined whether it is in a consumer's installation or in the mains themselves. In the latter case, the fault has sometimes been detected by noticing a dry patch on a wet pavement.

If the fault be on a neutral feeder, it can be readily detected by putting a few secondary cells or a booster in series with the earth ammeters in the A and B sections in turn. If there be a fault in the A section, it will be possible to get a large constant current when the cells are in series with the earth ammeter of the A section. Now remove the feeders from the A section to the B section in turn. The faulty feeder will be detected by its transference reducing very considerably the reading of the A ammeter.

The Hopkinson three ammeter method

In this method a section of the network is isolated and the readings of the ammeters on the positive, neutral, and negative feeders of this section are taken. If the sum of two of the readings be not equal to the third there must obviously be a leak on the given section of the network. It has to be noticed, however, that even when the sum of two of them is approximately equal to the third there may be a fault on the neutral if its potential is small. Hence we must test for a neutral leak by putting a few cells in circuit with the neutral and an earth connexion in the manner described above. This method can only be successfully used when the load is very steady. If there is a motor load on the section, it is exceedingly difficult to get consistent simultaneous readings of the three ammeters.

The final methods of localization

If the system be a "drawn in" system and the fault has been localized to a particular section, then, if the cables are lead covered and unbraided we can often by feeling the lead at the service boxes detect by the slight shock generally experienced, the portion of the cable in which the fault is situated. Sometimes, also, more especially with rubber and vulcanized bitumen cables, the fault can be localized by the smell of the overheated insulating material. If, however, the cables are "solid-laid," or are armoured and laid direct in the ground, one or other of the following methods, which have been found useful in practical work, can be used.

The fall of potential method

Let us suppose that the fault lies in the particular loop *LFM* (Fig. 41). Earth one pole of the battery and connect the other through a resistance *R*, and an ammeter *A*, with the end *L* of the main *LM*. As *ERLFE* forms a closed circuit, a current will flow which can be read on the ammeter. An electrostatic

voltmeter placed between L and M will read the voltage V between L and F, for F and M are at the same potential since there is no current in FM. If x be the resistance of LF, we have $x=V/A$, where A is the ammeter reading,

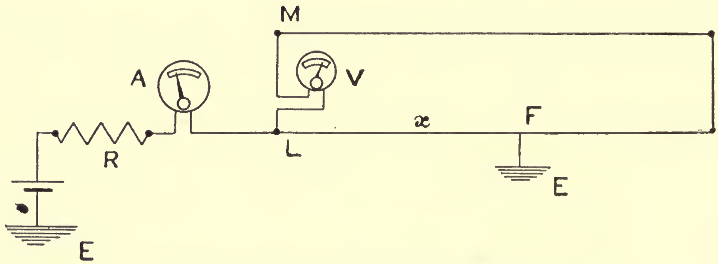

FIG. 41.—Fall of Potential Method.

and thus x is found. Knowing the resistance of a yard of cable the distance to the fault can now be readily calculated with considerable accuracy.

It is to be noticed that we have made the assumption that there is only one fault in the main. If there were more than one fault, this calculated length would be greater than the distance to the first fault. It is therefore always advisable to repeat the test, connecting the end M of the cable with the battery. If the two results obtained on the assumption that there is a single fault agree in locating this fault at the same point of the cable, it is highly probable that a fault will be found at this point.

When a spare drum of the same cable as the faulty main is available at the station, the following modification of the above test will be found convenient. Replace the ammeter A and the resistance R (Fig. 41) by the drum of cable, everything else remaining as before. Take the reading V' of the voltmeter when placed across the terminals of the drum and the reading V between L and M. Then if l' be the

length of the cable on the drum and l the distance LF to the fault we have $l=(V/V')l'$.

Loop test

The principle of the loop test can be readily understood from Fig. 42. A wire bridge pq is connected across the terminals L and M of the loop of cable in which the fault lies, and a galvanometer G is also placed between the terminals. One pole of a battery is connected with the jockey of the bridge, the other pole being earthed. The jockey is then moved about until the deflection of the galvanometer is zero. In this case we

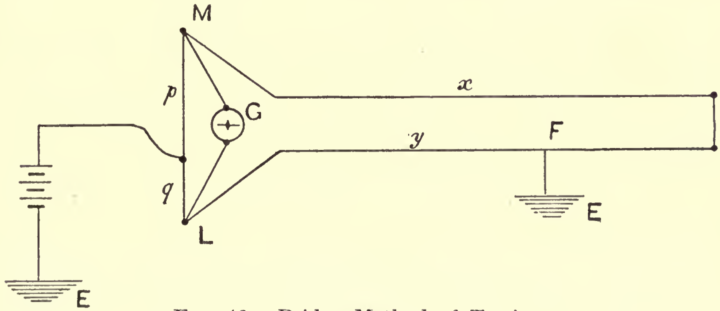

Fig. 42.—Bridge Method of Testing.

have $x/y=p/q$, and thus $y=\{q/(p+q)\}(x+y)$. Hence, if l be the length of the whole loop, the length of LF is $\{q/(p+q)\}l$.

In this test care has to be taken that the resistance of the connexions at L and M are negligibly small. It is immaterial whether the fault at F is polarizable or not as it is in series with the battery. This is one of the most generally useful and accurate of the methods of localizing faults.

Two ammeter method

In certain cases the following test can be easily applied. Place ammeters A_1 and A_2 (Fig. 43), of negligible resistance, in series with each branch of the loop. Now connect L with one pole

of a battery, the other pole of which is earthed. If A_1 and A_2 be the readings of the ammeters, then, by Ohm's law, the potential difference between L and F will equal xA_1, and it will also equal yA_2. Hence $y(A_1+A_2)=(x+y)A_1$, and thus, if l be the total length of the loop, the distance LF to the fault will equal $\{A_1/(A_1+A_2)\}l$.

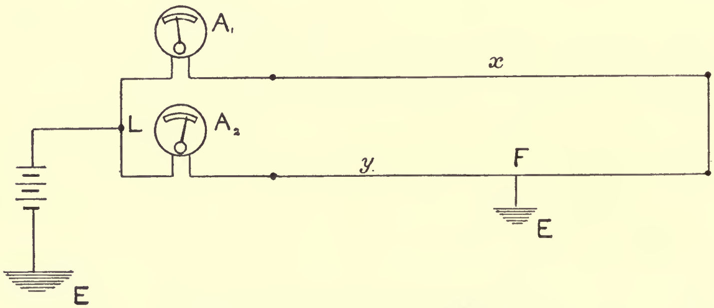

FIG. 43.—Two Ammeter Method.

In the above tests, it has been assumed that the section of the cable is uniform throughout. If it is not uniform, they can still be applied, provided the lengths and the resistances of the various portions of the cables are known. Let us suppose that the lengths LL_1, L_1L_2, ... are of different sections and that their resistances are R_1, R_2, ... respectively. Let us also suppose that the resistance y of LF lies in value between R_1+R_2 and $R_1+R_2+R_3$. The fault will obviously be on the section L_2L_3, and the resistance of the conductor between L_2 and the fault will be $y—(R_1+R_2)$. Hence, knowing the resistance per yard of the section L_2L_3 of the cable, we can easily find the position of the fault.

Induction method

The following "induction method" is extensively used in practice. The great advantage of this method is that no knowledge is required of

the resistance of the main under test, and so uncertainties due to partial breaks or bad joints do not affect it. It can also be applied when there are several earth faults on the cable.

Let us suppose that there is an earth fault at *F* in the cable *LM* (Fig. 44). Insulate one end *M* of the cable and

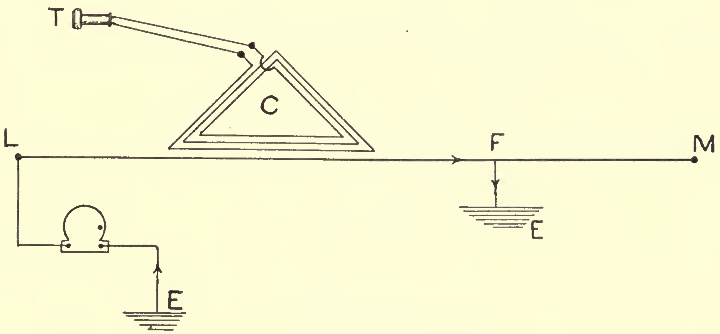

FIG. 44.—Induction Method.

connect the other end with the terminal of a generator of alternating or pulsating E.M.F. If the other end of this generator be connected with the earth, an interrupted current will flow in the cable to the fault and return to the generator by the earth. *C* is a wooden triangular framework wound with several turns of insulated wire the ends of which are connected with a telephone *T*. When the plane of the triangle is vertical and its base is parallel to the cable carrying the interrupted current the continual fluctuation of the lines of magnetic induction, linked with the triangular coil, will induce a fluctuating E.M.F. in it, and so a buzzing sound will be heard in the telephone receiver. If the framework, held in this manner, be carried or wheeled directly over and along the route of the cable, a cessation or change of the note generally indicates the position of the fault. If the conductor carrying the current

be enclosed in a lead sheath or in metallic casing, the earthed terminal of the generator should be connected with the sheath or the casing.

Blavier's test

When the resistance of the line is high, the following method is sometimes useful. Let us suppose (Fig. 45) that there is a fault of resistance z at F. Let the resistances also of LF and FM be x and y respectively. In practice, the resistance R of the whole line LM is known.

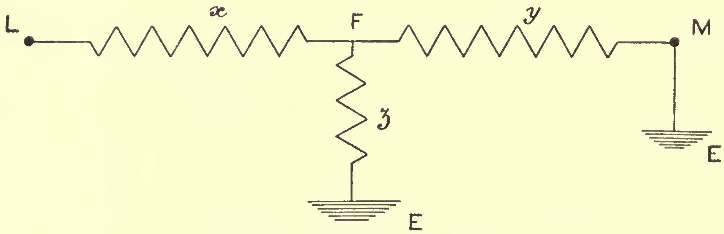

FIG. 45.—Blavier's Method.

We first measure at L the insulation resistance R_1 of the line, when the end M is insulated. We next measure its insulation resistance R_2, when the end M is connected directly with the earth. We thus obtain the three following equations to find x, y and z.

$$x+y=R \quad \ldots \quad (1),$$

$$x+z=R_1 \quad \ldots \quad (2),$$

and

$$x+yz/(y+z)=R_2 \quad \ldots \quad (3).$$

Eliminating y and z from (3), by means of (1) and (2), we have

$$x+(R-x)(R_1-x)/(R+R_1-2x)=R_2,$$

and thus, by solving this quadratic equation we get

$$x=R_2-\{(R-R_2)(R_1-R_2)\}^{1/2},$$

the negative sign being prefixed to the radical, since by the conditions of the problem $yz/(y+z)$ can only be positive, and hence, by (3), x must be less than R_2.

There are four assumptions made in this test that have to be remembered in practical work. In the first place we assume that there is only one fault, secondly that the fault is non polarizable, thirdly that the temperature of the mains is uniform, and fourthly that the resistances are so high that the errors due to imperfect contacts are negligibly small. In electric lighting networks the fourth assumption is the most serious and in many cases it is not permissible.

Example In a power transmission line, R was 44 ohms, and the values of R_1 and R_2 found by measurement were 25·9 and 24·3 respectively. In this case

$$x = 24{\cdot}3 - \{(25{\cdot}9 - 24{\cdot}3)(44 - 24{\cdot}3)\}^{1/2}$$
$$= 18{\cdot}7.$$

By (2), $18{\cdot}7 + z = 25{\cdot}9$,

and therefore, $z = 7{\cdot}2$.

Similarly, by (1), $y = 25{\cdot}3$.

When tests can be made at both ends of the line it is advisable to make them. If the results differ largely it is probable that there is more than one fault.

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DIELECTRIC STRENGTH

CHAPTER VIII

Dielectric Strength

Dielectric strength—Disruptive discharge—Spherical condenser—Single core main—Effect of shape of conductor—Equipotential surfaces round a pointed conductor—Spherical electrodes—Composite dielectrics—The maximum electric stress between equal spherical electrodes—American rules—Failing cases in practice—Measuring the dielectric strength of gases—Dielectric strength of liquids—Dielectric strength of isotropic solids—Dielectric strength of eolotropic solids—References.

Dielectric strength

In power transmission, whether by direct or alternating current, the saving of copper effected by using very high pressures has directed the attention of manufacturers to the construction of cables which will withstand pressures of 100 kilovolts and upwards. To design cables which will successfully withstand these pressures, a knowledge of the electric stresses to which the various insulating materials round the core will be subjected under working conditions is essential, as well as an accurate knowledge of the dielectric coefficient (specific inductive capacity), dielectric strength, and resistivity of each of the insulating wrappings.

In this chapter we shall discuss the dielectric strength of insulating materials and the methods of measuring it. By the dielectric strength of an isotropic insulating material in a given physical condition is meant the maximum value of the electric stress which it can withstand without break-

ing down. The substance of a homogeneous solid is called isotropic (*see* p. 3) when a spherical portion of it tested by any physical agency exhibits no difference in quality however it is turned. Substances which are not isotropic are called eolotropic. From an electrical point of view we can regard gases and pure liquids as isotropic.

In his *Experimental Researches in Electricity*, vol. i, p. 436, Faraday states that "discharge probably occurs not when all the particles have attained to a certain degree of tension, but when that particle which has been most affected has been exalted to the subverting or turning point." He therefore considers that there is a definite "subverting or turning point" for each particle of the material; that is, that it has a definite dielectric strength.

In order to test Faraday's conclusion it is necessary to be able to calculate the electric stress at the point between two electrodes where it has its maxiumm value. We shall first consider how electric stress is measured.

From symmetry, it is obvious that the equipotential surfaces round a charged spherical conductor surrounded by a homogeneous dielectric, and at a considerable distance from all other conductors, are spherical in shape. If q denote the charge on the conductor, and v the potential at a point at a distance r from the centre of the sphere, we have $v=q/r$, and therefore the potentials of the equipotential surfaces surrounding the sphere vary inversely as their radii. Let us suppose, for example, that the spherical conductor shown in Fig. 46 is at a potential of 10,000 volts, then the potentials of the various circles drawn in the figure are 9,000, 8,000, ... 1,000, volts respectively. The equipotential surface of zero potential would be at infinity. It is to be noticed that close up to the sphere the surfaces are crowded together. The

spherical stratum of the dielectric included between the conductor and the first equipotential surface is obviously subjected to the greatest stress. The average stress on any of the spherical layers shown in Fig. 46 is inversely proportional to its thickness. By increasing the number of equipotential surfaces indefinitely until the concentric

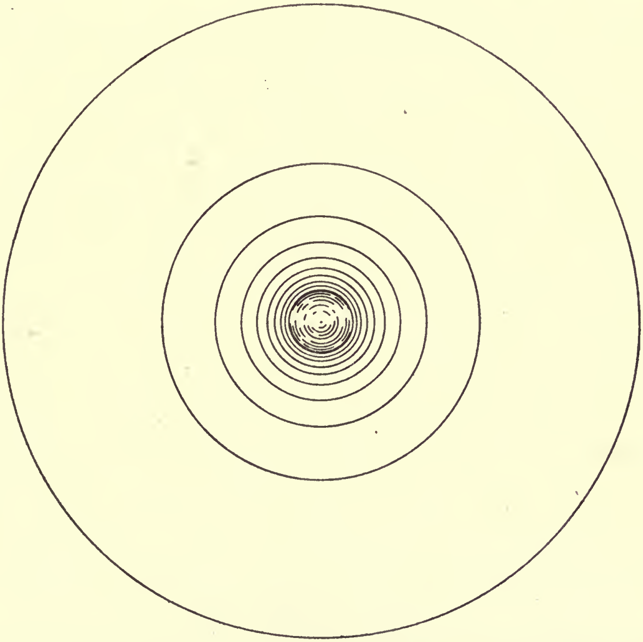

Fig. 46.—Section of the equipotential surfaces round a charged sphere, the potential difference between consecutive surfaces being constant.

layers of the dielectric become indefinitely thin, we see that the electric stress at a point is measured by $\{v-(v+dv)\}/dr$, that is, by $-dv/dr$. This quantity is called by electricians the potential gradient at the point.

It follows at once from the definition of potential that the resultant force at a point in a dielectric is equal to the rate at which the potential diminishes as we move

along the line of force through the point. Hence the potential gradient is the resultant force, and is the force with which a unit positive charge placed at the point would be urged if it could be placed there without disturbing the distribution elsewhere. This force measures the electric stress on the dielectric.

A good way of picturing what happens in a dielectric is by means of Faraday's tubes of force. We picture one end of one of these tubes anchored to a unit positive charge on the positive electrode, and the other end anchored to a unit negative charge on the other electrode.

In the case of the spherical conductor considered above, q tubes will start from the surface of the sphere, and thus the number of tubes passing through a square centimetre of the equipotential surface which is at a distance r from the centre of the sphere is $q/4\pi r^2$. But the potential gradient at a distance r from the centre of the sphere is $-q/r^2$, and hence 4π times the number of Faraday tubes which pass through unit area of the equipotential surface is the numerical value of the potential gradient at all points on that surface. Another name for the electric stress at a point is the electric "intensity" at the point. It has to be remembered when reading the literature of the subject that, "the resultant electric force," "the potential gradient" and "the electric intensity" are all used to denote the resultant electric stress R at a point. In symbols,

$$R = -dv/dr = 4\pi N.$$

If R were constant over the equipotential surface passing through the point under consideration, N would be the number of Faraday tubes per square centimetre of this surface.

Disruptive discharge

Greatly to the disappointment of the earlier physicists, it was found that the disruptive voltage between metal electrodes when close together

was apparently not governed by the maximum value of the potential gradient between them. As early as 1860, however, Lord Kelvin was led to infer, from his experiments with pressures of between 5 and 6 kilovolts obtained from a battery of 5510 Daniell cells in series, that at high voltages the disruptive discharge between large metal electrodes will take place the moment the electric stress attains a definite value. Recent experiments at high voltages have amply confirmed Lord Kelvin's conclusion, and it forms the basis of the practical methods of measuring dielectric strengths.

It has to be remembered that part of an insulating material can break down without a disruptive discharge necessarily ensuing. When brush discharges, for instance, occur from an electrode in air part of the air surrounding the electrode has become a true gaseous electrolyte, and its insulativity therefore has broken down. The air at the boundary of this electrolyte has not broken down, because the electric stress to which it has been subjected has not reached the "subverting" value, which measures the dielectric strength of the medium.

In Fig. 46, when the stress close up to the sphere equals the dielectric strength of air, which is about 38 kilovolts per centimetre at ordinary temperatures and pressures, the spherical layer round it breaks down and becomes luminous. If we raise the potential still higher the sphere is surrounded by a luminous spherical envelope called the corona, the radius of which is proportional to the potential to which we raise the conductor.

Spherical condenser

In the case of a spherical condenser we have a metallic sphere concentric with a metallic spherical envelope. If the radius of the inner sphere be a, and the radius of the outer sphere be b, we have $v=q/r$,

and $q=Vab/(b-a)$, where V is the potential difference between the two conductors. The equipotential surfaces are thus the same as in Fig. 46, and since

$$R=-dv/dr=Vab/r^2(b-a),$$

we see that R has its maximum value R_m, when $r=a$. Hence $R_m=Vb/a(b-a)$.

If we suppose that the sphere is surrounded by air, then, when R_m attains the value of the dielectric strength of air $R_{max.}$, the air surrounding the sphere breaks down and becomes a conductor, but a disruptive discharge does not necessarily ensue. If the breaking down of the first stratum of air makes the new value of R_m equal to or greater than $R_{max.}$, a disruptive discharge will ensue but if it makes it less than $R_{max.}$ there will be no disruptive discharge, and a corona will be formed.

If a_1 be the radius of the corona formed, we have

$$V=a_1(b-a_1)R_{max.}/b,$$

and thus $$dV/da_1=(b-2a_1)R_{max.}/b,$$

assuming that $R_{max.}$ and b are constant. We see that V increases with a_1 until a_1 gets equal to $b/2$, it then diminishes as a_1 increases. Hence a corona can exist if a_1 be less than $b/2$, for the value of R_m in the stratum immediately surrounding the corona is less than $R_{max.}$ It cannot, however, exist if a_1 be greater than $b/2$, for the value of R_m in the stratum surrounding it would be greater than $R_{max.}$ We see, therefore, that the size of the inner sphere has no practical effect on the disruptive voltage provided that its radius be less than $b/2$. We see also that a spherical condenser can be used to measure the dielectric strengths of gases or liquids provided that the radius of the inner conductor be not less than $b/2$. In this case

$$R_{max.}=bV/a(b-a),$$

where V is the voltage between the conductors at the instant of the discharge.

Single core main

In Fig. 47 the equipotential surfaces for a single core cable with a homogeneous dielectric are shown. Let us suppose that a is the radius of the

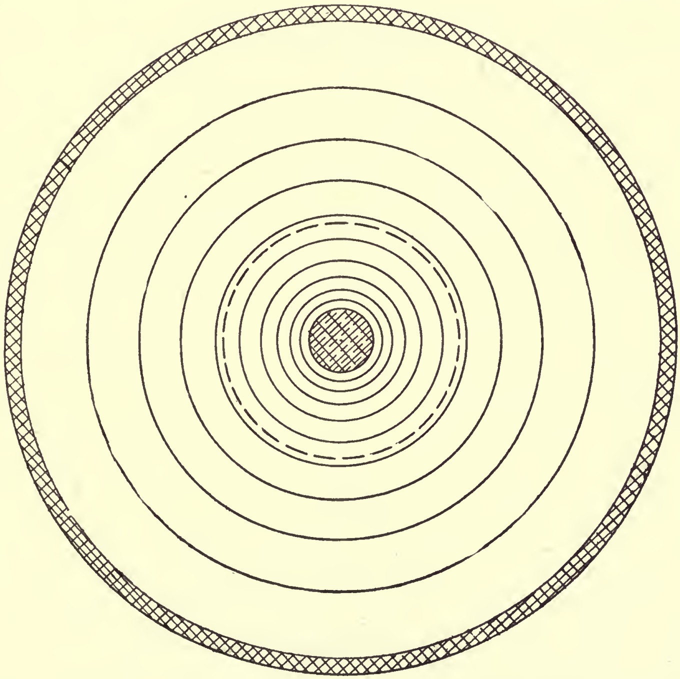

FIG. 47.—Section of the equipotential surfaces in a single core cable. The dotted circle is the outer radius of the broken-down dielectric at the instant of the disruptive discharge.

cylindrical core, and that b is the inner radius of the coaxial cylindrical lead sheath. The potential gradient R at a point P in the dielectric which is distant r from the axis of the core, is given by

$$R = -(1/\lambda)dv/dr = V/r\log_\epsilon(b/a),$$

where V is the potential difference between the core and

the sheath, and λ is the dielectric coefficient of the insulating material. R has its maximum value R_m when $r=a$, and thus $R_m = V/a\log_\epsilon(b/a)$.

Let us first suppose that the insulating material is a gas of dielectric strength $R_{max.}$. The conditions that a cylindrical corona of radius a_1 be formed are that

$$V = a_1 \log_\epsilon(b/a_1) \,.\, R_{max.},$$

and that dv/da is a positive quantity when $a=a_1$. The second condition is true when $\log_\epsilon(b/a_1) - 1$ is positive, that is, when a_1 is less than b/ϵ, where $\epsilon = 2{\cdot}718 \ldots$ is the base of Neperian logarithms.

When the insulating material is a homogeneous solid substance of dielectric strength $R_{max.}$ the same formulæ apply, at least to a first approximation. If the radius of the core be less than b/ϵ, then, when V is greater than $a\log_\epsilon(b/a)R_{max.}$, and less than $(b/\epsilon)R_{max.}$, some of the dielectric surrounding the inner core, which we suppose to be a smooth cylinder, has broken down and become a conductor. When the voltage V exceeds $(b/\epsilon)R_{max.}$ a disruptive discharge will ensue.

A comparison of Figs. 46 and 47 will show that the electric stresses close to a spherical conductor are greater than close to a cylindrical conductor of the same radius and at the same potential. It will be noticed that the equipotential surfaces are more crowded together round the sphere than round the cylinder. Since we have supposed the cylinder to be infinitely long, the change of potential as we recede from it will obviously not be so rapid as in the case of the finite sphere.

Effect of shape of conductor

In Fig. 48 the effect of the shape of a conductor on the electric stresses in the medium surrounding it is illustrated. In the figure the potential difference between any adjacent pair of

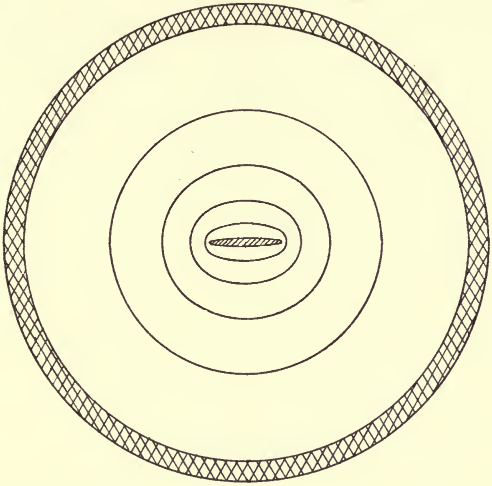

FIG. 48.—Section of the equipotential surfaces when the core is a strip of copper with rounded ends.

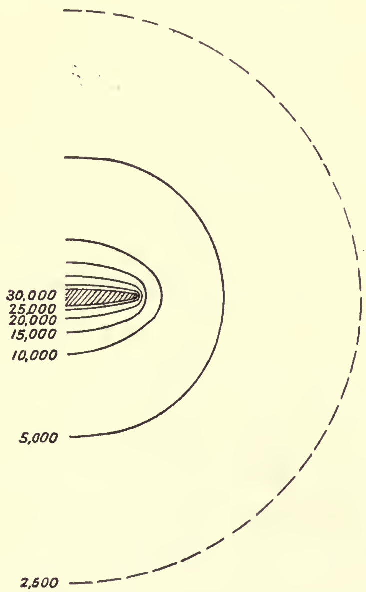

FIG. 49.—Section of the equipotential surfaces round a tapered copper conductor maintained at 30 kilovolts.

equipotential surfaces is the same. The section of the core is elliptical in shape, and the maximum value of the electric stress at the rounded ends of the core is ten times the minimum stress at the middle part of the core. The equipotential surfaces show, however, that the electric stress is practically constant in the layer of the insulating material next the lead sheath.

Equipotential surfaces round a pointed conductor

In Fig. 49, the equipotential surfaces round a tapered copper conductor, ellipsoidal in shape, are shown. The electric stress on the insulating material in contact with the rounded point is very great. When electrodes of this shape are used for testing it is extremely difficult to calculate

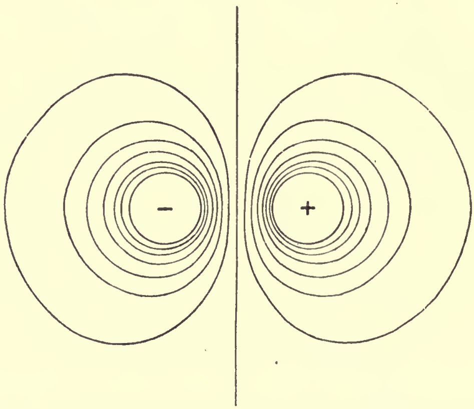

Fig. 50.—Section of the equipotential surfaces round two spheres having equal and opposite charges. The potential gradient in the dielectric is obviously greatest at the points of the electrodes which are closest together.

the value of the maximum potential gradient, and hence, only rough comparative tests can be made with them.

Spherical electrodes

In Fig. 50, the equipotential surfaces round two spherical electrodes maintained at equal and opposite potentials are shown. The potential difference

between any two adjacent surfaces is the same. The potential gradient is obviously a maximum at the points of the electrodes which are closest together. The value of the potential gradient at these points can be easily calculated by means of the tables given below. It can be shown that if the spheres be not farther apart than twice their diameter, a disruptive discharge will take place the moment the portions of the insulating material which are

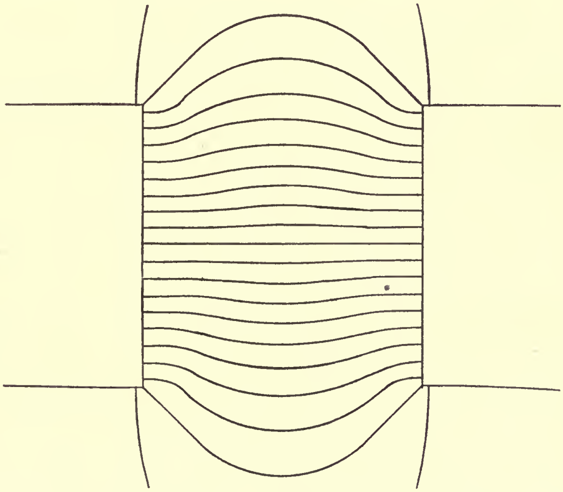

FIG. 51.—Flux lines between cylindrical electrodes. The potential gradient is a maximum at the corners of the electrodes.

subjected to the maximum stress break down. Hence, the disruptive voltage enables us to find the dielectric strength of the medium by which they are surrounded. For practical testing, spherical electrodes are generally the best.

Composite dielectrics

The effect of introducing a piece of insulating material between two metal electrodes maintained at constant potentials is illustrated in Figs. 51 and

52. The insulating material is supposed to have a high dielectric coefficient. The capacity and consequently the number of Faraday tubes between the electrodes is considerably increased. The stress on the air which is measured by 4π times the number of Faraday tubes per unit area is increased also. Hence the introduction of a piece of glass near the electrodes sometimes causes a disruptive discharge

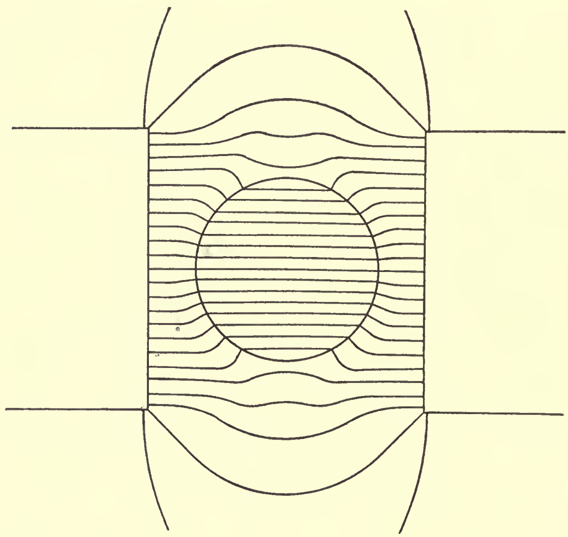

Fig. 52.—Flux lines when a glass sphere is introduced between the electrodes, the potential difference being maintained the same as in Fig. 51. Notice the increase in the total flux, and consequently the increase in the stress on the dielectric.

between them. Since the dielectric coefficient of a metal is infinite, introducing a metal conductor between the electrodes increases the stress more than a piece of insulating material of the same size would. The calculation of the maximum potential gradient when the insulating materials have different dielectric coefficients is in general very difficult.

The maximum electric stress between equal spherical electrodes

The easiest way of finding the dielectric strength of insulating materials is by finding the disruptive voltage between two equal spherical electrodes embedded in the material. The author has shown (*Phil. Mag.* (6), vol. ii, p. 258, 1906), that, if the spheres be at a less distance apart than twice the diameter of either, a disruptive discharge will ensue the moment the maximum electric stress between the spheres equals the dielectric strength of the material. In order to calculate the maximum electric stress at the instant of discharge we must know the potential and size of each sphere and the distance between them.

Let a be the radius of each sphere, and let x be the minimum distance between them. Let us first suppose that one sphere is at the potential V_1 and that the other is at zero potential. In this case the maximum electric stress, $R_{max.}$, between them is given by

$$R_{max.} = (V_1/x) f_1,$$

where the values of f_1 can be found from Table II. A proof of this formula is given in the author's paper (quoted above). In the important practical case when $V_1 = -V_2 = V/2$, where V_2 is the potential of the second sphere, we have

$$R_{max.} = (V/x) f,$$

where f can be found from Table I.

In general, if V_1 and V_2 be the potentials of the two spherical electrodes, and V_1 be numerically greater than V_2, we have

$$R_{max.} = \{(V_1 - V_2)/x\} f_1 + 2(V_2/x)\ (f_1 - f),$$

where f and f_1 are functions of x/a, the values of which can be found from Tables I and II.

Hence by this formula we can calculate the dielectric

strength of the material from the potentials of the electrodes at the instant of the disruptive discharge.

TABLE I.

VALUES OF f.

x/a.	f.	x/a.	f.
0·0	1·000	2	1·770
0·1	1·034	3	2·214
0·2	1·068	4	2·677
0·3	1·102	5	3·151
0·4	1·137	6	3·632
0·5	1·173	7	4·117
0·6	1·208	8	4·604
0·7	1·245	9	5·095
0·8	1·283	10	5·586
0·9	1·321	100	50·51
1·0	1·359	1,000	500·5
1·5	1·559	10,000	5,000·5

TABLE II.

VALUES OF f_1.

x/a.	f_1.	x/a.	f_1.
0·0	1·000	2	2·339
0·1	1·034	3	3·252
0·2	1·068	4	4·201
0·3	1·106	5	5·167
0·4	1·150	6	6·143
0·5	1·199	7	7·125
0·6	1·253	8	8·111
0·7	1·313	9	9·100
0·8	1·378	10	10·091
0·9	1·446	100	100·0
1·0	1·517	1,000	1,000
1·5	1·909	10,000	10,000

The dielectric strength of air

In the following table the values of x and V are taken from Dr. Zenneck's work, *Elektromagnetische Schwingungen und Drahtlose Telegraphie*, 1905, p. 1011. They are due to J. Algermissen, and are deduced from the average of the values obtained on different days under varying conditions. It has been assumed that the potentials of the electrodes are $+V/2$ and $-V/2$ respectively at the instant of discharge. As the results in the last column are very approximately constant the assumption is justified :—

TABLE III.

J. Algermissen. 5-cm. spheres ($a=2{\cdot}5$). x is measured in centimetres, and V in kilovolts.

x.	x/a.	f (calc.).	V (obs.).	R_{max} (calc.).
2·0	0·80	1·283	58·2	37·3
2·2	0·88	1·312	62·8	37·5
2·4	0·96	1·342	67·0	37·5
2·6	1·04	1·374	70·8	37·4
2·8	1·12	1·406	74·4	37·4
3·0	1·20	1·437	78·0	37·4
3·2	1·28	1·469	81·3	37·3
3·4	1·36	1·500	84·7	37·4
3·6	1·44	1·533	88·0	37·5
3·8	1·52	1·566	91·2	37·6
4·0	1·60	1·599	94·2	37·7
4·2	1·68	1·632	97·2	37·5

In the above table $R_{max.}$ has been calculated by means of the formula

$$R_{max.}=(V/x)f.$$

From the above results, and from many other experimental results obtained with both alternating and direct pressures, the author concludes that the dielectric strength

of air under normal conditions is about 3·8 kilovolts per millimetre.

American rules

The practical constancy of the dielectric strength of air under ordinary atmospheric conditions is recognized in the Standardization Rules (1907) of the A.I.E.E. For instance, in § 243, when discussing the value of the spark-gap safety-valve, it is stated that "a given setting of the spark-gap is a measure of one definite voltage, and, as its operation depends upon the maximum value of the voltage wave, it is independent of wave form, and is a limit on the maximum stress to which the insulation is subjected. The spark-gap is not conveniently adapted for comparatively low voltages." The reason for the limitation given in the last sentence of the above quotation will be discussed below.

In Appendix D of the American Rules, the following table of the sparking distances in air between "opposed sharp needle-points" for sine-shaped voltage waves is given. The numbers are calculated from the experimental results given in a paper on the "Dielectric Strength of Air," by Professor Steinmetz, published in the *Transactions of the American Institute of Electrical Engineers*, vol. xv, p. 281. Under normal atmospheric conditions, with new sewing needles supported axially at the end of linear conductors which are at least twice the length of the gap, the maximum difference between the observed disruptive voltages and the values given in the table will probably be well within 5 per cent. of either voltage. Care must be taken that the potentials of the needles are always equal and opposite and that no foreign body is near the spark-gap. In practical work it is also important that a non-inductive resistance of about one-half of an ohm per volt should be inserted in series with each terminal of the gap

so as to keep the discharge current between the limits of one-quarter ampere and two amperes. The object of limiting this current is to prevent the surges of the voltage and current which might otherwise occur at the instant of breakdown.

TABLE IV.

SPARKING DISTANCES BETWEEN NEEDLE POINTS.

Effective Kilovolts.	Inches.	Cms.	Effective Kilovolts.	Inches.	Cms.
5	0·225	0·57	140	13·95	35·4
10	0·47	1·19	150	15·0	38·1
15	0·725	1·84	160	16·05	40·7
20	1·00	2·54	170	17·10	43·4
25	1·30	3·3	180	18·15	46·1
30	1·625	4·1	190	19·20	48·8
35	2·00	5·1	200	20·25	51·4
40	2·45	6·2	210	21·30	54·1
45	2·95	7·5	220	22·35	56·8
50	3·55	9·0	230	23·40	59·4
60	4·65	11·8	240	24·45	62·1
70	5·85	14·9	250	25·50	64·7
80	7·10	18·0	260	26·50	67·3
90	8·35	21·2	270	27·50	69·8
100	9·60	24·4	280	28·50	72·4
110	10·75	27·3	290	29·50	74·9
120	11·85	30·1	300	30·50	77·4
130	12·90	32·8			

Failing cases in practice

When spark-gaps between needle points are used to measure very low voltages very unsatisfactory results are obtained. Even when the electrodes are spherical it is very difficult to obtain consistent results when the distance between them is less than a millimetre. When the electrodes are at microscopic distances apart the above formulae cannot be applied in practice. G. M. Hobbs (*Phil. Mag.* [6], 10, p. 617) has shown that when the minimum distance x between the

spheres is less than 3μ, where $\mu = 10^{-6}$ metre, the sparking potentials are practically independent of the nature of the gas between the electrodes. They depend, however, on the metal of which the electrodes are made. When the electrodes are very close together, it has to be remembered that our assumption of an isotropic medium bounded by smooth rigid equipotential surfaces is no longer permissible. If the surfaces were magnified sufficiently they would be seen to be rough, and the dielectric surrounding the microscopic projections would probably be ionized. In these circumstances, therefore, accurate calculations would be difficult.

Hence, in determining dielectric strengths, it is necessary to have the electrodes at appreciable distances apart, and therefore high voltages must be used. It is not safe to calculate dielectric strengths from the observed disruptive voltages when the electrodes are less than a millimetre apart. When a maximum inaccuracy of more than 1 per cent. is not permissible, they should be at least half a centimetre apart.

It has also to be remembered that the formulae for the maximum value of the electric stress on the medium between spherical electrodes have been obtained on the assumption that the Faraday tubes are in statical equilibrium. In the case of impulsive rushes of electricity (*see* Chapter XII), or with alternating pressures at exceedingly high frequencies, the disruptive voltages seem to be independent of the shape of the electrodes.

Measuring the dielectric strength of gases

The dielectric strength of a gas may be deduced from experiments on the sparking voltages between spherical electrodes. The containing vessel for the gas should be large with the spherical electrodes near the centre. The dia-

meter of the supporting rods should be small compared with the diameter of the electrodes, and care should be taken that no conducting materials or insulating materials having dielectric coefficients different from the gas are in the immediate vicinity of the electrodes, otherwise the distribution of the Faraday tubes between the electrodes will be altered and our formulæ will not apply. It is usually best to earth the middle point of the secondary coil of the transformer, or the middle point of the batteries used, so as to make the potentials of the electrodes equal and opposite at the instant of discharge.

If $E/2$ and $-E/2$ be the potentials of the electrodes, at the instant of discharge, when direct voltages are used, we have

$$R_{max.}=(E/x)f,$$

where $R_{max.}$ is the dielectric strength of the gas, x the minimum distance between the electrodes, and f a number which can be obtained from Table I. The nearest points on the electrodes should not be closer than about half a centimetre, and their diameter should be about 5 cm. With air at atmospheric pressure a voltage slightly less than 20 kilovolts would be required when x was 0·5 cm.

When alternating pressures are used it is absolutely necessary to know the ratio of the maximum voltage E to the effective voltage V. Let this ratio, which is sometimes called the amplitude factor, be denoted by k, then our formula is

$$R_{max.}=(kV/x)f.$$

Steinmetz's method of putting the electrodes into nitrate of mercury, and rubbing them with a clean cloth, before and during the experiments is to be commended. This is especially necessary when the electrodes are only a small distance apart.

The pressure, temperature, and humidity of the gas must be given.

J. N. Collie and W. Ramsay (*Proc. Roy. Soc.*, vol. lix., p. 257, 1896) give interesting comparative values of the sparking potentials for various gases contained in glass tubes. The electrodes were of platinum with slightly rounded ends. Owing to the dielectric coefficient of the glass tube being different from that of the gas, and owing to the great electric stress at the electrodes causing excessive ionization, absolute values cannot be found from their results, but the following table shows that the dielectric strengths of the gases differ considerably :—

Gas.	Sparking Distances in mms.
Oxygen	23
Air	33
Hydrogen	39
Argon	45·5
Helium	greater than 250.

The dielectric strength of helium, therefore, is extraordinarily low compared with that of other gases.

Dielectric strength of liquids

The liquid to be tested is generally placed in a vertical glass cylinder about 2 in. in diameter. Spherical electrodes about half an inch in diameter are immersed in the liquid, and the distance between them is varied by means of a micrometer screw. The formulae for deducing the dielectric strength from the disruptive voltage are the same as for a gas.

The electrodes should not be less than 0·3 of a centimetre apart, and at this distance 40 or 50 kilovolts will be required to break down good insulating oils. In some cases when water is present much smaller voltages suffice.

In order to find the true dielectric strength of an oil, it

is necessary to thoroughly dry it before the test. This can be done by letting hot air bubble up through it. It is inadvisable, however, to heat the oil above 100° C. as considerable discolouration often results and its physical state alters. When oils are dried in this way perfectly consistent results can be obtained.

As a numerical example, let us suppose that the disruptive voltage for an oil between 1 cm. spherical electrodes, 0·3 of a centimetre apart, is 28 kilovolts, V_1 being equal to $-V_2$, and the amplitude factor being 1·5. By Table I, we get

$$R_{max.} = (1{\cdot}5 \times 28/0{\cdot}3) \times 1{\cdot}21$$
$$= 168 \text{ kilovolts per centimetre.}$$

Dielectric strength of isotropic solids

If the spherical electrodes can be entirely embedded in the insulating material then we can proceed as for liquids and gases, the same formulae being employed.

The method frequently adopted of putting thin sheets of the insulating material between metal electrodes in air is of doubtful value. As the voltage is increased the air surrounding the electrodes is broken down long before the disruptive voltage is reached. The insulating material heats excessively, and the maximum electric stress to which it is subjected cannot be calculated as the temperature is rarely uniform throughout, and the insulativity of the medium and the dielectric coefficient vary with the temperature. Results obtained by neglecting the variations of the electric stress due to temperature are useful only when all the conditions of the experiment are mentioned.

Dielectric strength of eolotropic solids

When the insulating material is eolotropic the calculation of the electric stresses is very difficult. They vary with the dielectric coefficients and the insulativities of the various constituents,

and, as we have just mentioned, these quantities vary rapidly with the temperature. Accurate measurements of the mean dielectric strength are therefore in many cases almost impossible.

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THE GRADING OF CABLES

CHAPTER IX

The Grading of Cables

The grading of cables—Concentric main—Suitable dimensions for a concentric main—Grading single core cables for alternating pressures—Grading single core cables for direct pressures—Jona's graded cables—The effects of leakage currents on the grading of concentric mains—Numerical example—The British standard radial thicknesses for jute and paper dielectrics—The effects of stranding on the electric stresses—Conclusions—References.

The grading of cables By the grading of cables is meant the arranging of the order and thickness of properly chosen insulating wrappings so that each bears its due share of the total electric stress to which the insulating material is subjected. In addition, the electric stress on every wrapping must be as uniform as possible throughout its substance. We shall see that it is only possible to secure absolute uniformity of stress in a wrapping by making the dielectric coefficient of the insulating material diminish in a regular manner as we proceed outwards from the axis of the cable.

In the special case of a single core main with a homogeneous dielectric, the maximum electric stress $R_{max.}$ occurs next the core, and the minimum $R_{min.}$ next the lead sheath. If a be the radius of the core, the surface of which we suppose to be smooth, and b the inner radius of the lead sheath, we have $R_{max.} = V/a \log_\epsilon(b/a)$, and $R_{min.} = V/b \log_\epsilon(b/a)$, where V is the potential difference between the core and the lead sheath. We see that in this case $R_{max.}/R_{min.} = b/a$,

and if b/a be large, the material next the core will have to withstand a stress much greater than the average stress, which equals $V/(b-a)$. The layer of insulating material, therefore, has to be made very much thicker than if it had merely to insulate from one another two infinite plane surfaces at the given potentials. If it were possible to make the electric stress on the dielectric of a single core main uniform throughout so that its value was $V/(b-a)$, the thickness of the layer would be considerably reduced, and a considerable economy could be effected by the smaller amount of sheathing and armouring required.

We shall show how this can be done by using insulating materials having different dielectric coefficients and arranged in concentric wrappings of suitable thicknesses, and in a given order. Before doing this, however, we shall discuss the formula $R=V/\{x\log_\epsilon(b/a)\}$ for a concentric main, where R is the electric stress at points the distance of which from the axis of the cable is x. A proof of this formula is given in the author's treatise on *Alternating Currents*, vol. i, p. 95.

Concentric main

The formula shows us that the value of R at a point in the dielectric is independent of the absolute values of the potentials of the mains, and depends merely on the difference of the potentials and the distance of the point from the axis. We obviously have

$$R_{max.}=V/\{a\log_\epsilon(b/a)\}.$$

If V and b remain constant—

$$\frac{d\,R_{max.}}{d\,a}=\frac{V}{\{a\log_\epsilon(b/a)\}^2}\{1-\log_\epsilon(b/a)\}.$$

Hence, if a be less than b/ϵ where ϵ is the base of Neperian logarithms, $R_{max.}$ will diminish as a increases. In this case, we see that the breaking down of the dielectric round the inner core actually diminishes the maximum stress to which the dielectric is subjected. It is only when the radius of

the charred dielectric gets greater than b/ϵ that a disruptive discharge ensues.

Jona (*Trans. Int. Cong., St. Louis*, vol. ii, p. 550) describes an experiment on the disruptive voltages of two single-core cables of very different diameters, but each wound with the same thickness (1·4 cm.) of paper insulation. The core of one consisted of a thin wire 0·1 cm. in diameter, while the other was a copper cylinder 2·9 cm. in diameter. The former broke down at 40 kilovolts, and the latter at from 75–80 kilovolts. The former also got exceedingly hot after being subjected to 30 kilovolts for an hour, whilst the latter was still cold after 50 kilovolts had been applied for the same time. If we calculate the maximum electric stress on the dielectric surrounding the thin wire, on the assumption that no part of it is broken down before the disruptive discharge ensues, we get—

$$R_{max.} = \frac{40}{0{\cdot}05 \log_\epsilon(145/0{\cdot}05)}$$

$$= 238 \text{ kilovolts per centimetre.}$$

Similarly, the experimental results with the thick cable make $R_{max.}$ lie between 76·5 and 81·6 kilovolts per centimetre. This experiment is quoted by Jona to show that the ordinary formula cannot be applied when b/a is large.

If, however, we assume that the disruptive discharge does not occur until the outer radius of the charred dielectric becomes equal to b/ϵ, the experiment on the thin wire gives us—

$$R_{max.} = \frac{40 \times 2{\cdot}718}{1{\cdot}45}$$

$$= 75 \text{ kilovolts per centimetre, nearly,}$$

which, being in substantial agreement with the results given by the test on the thick cable, is a striking confirmation of the theory outlined above.

Suitable dimensions for a concentric main

Let us suppose that the maximum working voltage V, the density of the current in the inner conductor, and the maximum permissible stress to which the dielectric may be subjected, are fixed. Let us first suppose that the inner cylindrical conductor is solid and that its radius is a. If, then, V/l be the maximum permissible stress, we have

$$\frac{V}{a\log_\epsilon(b/a)}=\frac{V}{l},$$

and thus,

$$b=a\epsilon^{l/a}.$$

Hence also,

$$\frac{d\,b}{d\,a}=\epsilon^{l/a}\left(1-\frac{l}{a}\right).$$

If, therefore, a be greater than l, $d\,b/d\,a$ is positive, and therefore b increases as a increases, but if a be less than l, b diminishes as a increases. In the latter case it would obviously be advantageous to make the inner conductor hollow, its section remaining constant, so as to increase the value of a and diminish the value of b. The quantity of armouring and insulating material used would be diminished by this procedure. We conclude, therefore, that if a solid inner conductor of the required cross-section would have a radius less than l, the inner conductor should be made hollow and its outer radius should not be less than l. In some cases it would be advantageous to make the inner conductor of aluminium.

Although the inner radius of the outer conductor begins to increase when a gets greater than l, the following reasoning shows that the quantity of the dielectric required diminishes until a gets greater than $1{\cdot}25\,l$.

Using the same notation, the area of the cross-section of the dielectric of the cable is $\pi\,(b^2-a^2)$, and we have to

find the value of a that makes a^2 ($\epsilon^{2l/a}$—1) a minimum. Differentiating with respect to a and equating to zero, we get

$$\epsilon^{2l/a} = a/(a-l).$$

Let $a = nl$, then

$$2/n = \log_\epsilon n - \log_\epsilon(n-1).$$

By trial, we find that $n = 1{\cdot}2550$ . . . satisfies this equation, and hence, when n has this value the quantity of insulating material required is a minimum. In this case $a = 1{\cdot}255l$, $b = 2{\cdot}784l$, and $b = 2{\cdot}218a$. As the saving of insulating material effected by increasing a from l to $1{\cdot}25\ l$ is only about 3 per cent. it is of little importance compared with the increased cost of the armouring.

We conclude, therefore, that high-pressure concentric cables, having isotropic dielectrics, for use at a maximum voltage V, should be constructed so that $b = a\epsilon^{l/a}$, where V/l is the maximum permissible working stress to which the dielectric may be subjected, and a should never be made less than l.

Grading single core cables for alternating pressures

We shall first make the supposition that all the insulating wrappings used have the same dielectric strength, and that the maximum and minimum stresses to which they are subjected, when working, are to be the same for them all. We shall also suppose that the leakage current across the dielectric can be neglected in comparison with the capacity current. Let us suppose that there are n insulating wrappings the inner radii of which are a, r_2, r_3, . . . r_n, respectively, where a is the outer radius of the lead tube encasing the inner core, and let b equal the inner radius of the lead sheath. Since the ratio of the maximum to the minimum electric stress is to be the same in all the wrappings, we must have

$$\frac{r_2}{a} = \frac{r_3}{r_2} = \ldots = \frac{b}{r_n}.$$

We see, therefore, that the radii should be in geometrical progression, the common ratio being $(b/a)^{1/n}$. The thicknesses of the layers also form a geometrical progression having the same ratio $(b/a)^{1/n}$.

Let V_1, V_2, ... V_{n+1}, be the potentials of points at distances a, r_2, ... b from the axis of the cable. Then, since the layers form n condensers in series, the potential difference across a layer will be inversely proportional to the capacity of the layer, and thus we have

$$\frac{V_1-V_2}{(1/\lambda_1)\log_\epsilon(r_2/a)}=\frac{V_2-V_3}{(1/\lambda_2)\log_\epsilon(r_3/r_2)}=\ldots=\frac{V_n-V_{n+1}}{(1/\lambda_n)\log_\epsilon(b/r_n)}.$$

Hence, since the P.D.s are all in phase, each of these ratios equals $V\{\Sigma(1/\lambda_m)\log_\epsilon(r_{m+1}/r_m)\}$, where V is the voltage applied between the core and the sheath.

If R_m denote the maximum electric stress on the mth layer, we have

$$R_m=\frac{V_m-V_{m+1}}{r_m\log_\epsilon(r_{m+1}/r_m)}$$
$$=(V/\lambda_m r_m)/\Sigma(1/\lambda_m)\log_\epsilon(r_{m+1}/r_m).$$

Now, since the maximum stress on every layer is to be the same, we must arrange so that

$$\lambda_1 a=\lambda_2 r_2=\ldots=\lambda_n r_n.$$

Therefore

$$\frac{\lambda_1}{\lambda_2}=\frac{\lambda_2}{\lambda_3}=\ldots=\frac{\lambda_{n-1}}{\lambda_n}=\left(\frac{b}{a}\right)^{1/n}.$$

Hence λ_1, λ_2, ... λ_n, are the terms of a geometrical progression whose common ratio is $(b/a)^{1/n}$.

If $R_{max.}$ denotes the maximum electric stress in the graded cable, we have

$$R_{max.}=\frac{V}{a}\Big/\left(1+\frac{\lambda_1}{\lambda_2}+\frac{\lambda_1}{\lambda_3}+\ldots\right)\log(b/a)^{1/n}$$
$$=\frac{V}{a}\Big/\frac{b/a-1}{n\{(b/a)^{1/n}-1\}}\log(b/a)$$
$$=R'_{max.}\frac{n\{(b/a)^{1}-1\}}{b/a-1},$$

where $R'_{max.}$ stands for $V/a \log(b/a)$ the maximum stress in a cable of the given dimensions with an isotropic dielectric. If $R_{min.}$ denote the minimum electric stress in the dielectric of the graded cable, we have

$$R_{min.} = R_{max.}(a/b)^{1/n}.$$

In the ideal cable n would be infinite, and thus the stress would be the same at all points, and would equal $V/(b-a)$.

The capacity per unit length of a single-core cable, with isotropic dielectric equals $\lambda/\{2\log_\epsilon(b/a)\}$. The capacity of the graded cable equals $\lambda_1 n\{(b/a)^{1/n}-1\}/\{(b/a-1)2\log_\epsilon(b/a)\}$. When n is infinite this equals $\lambda_1 a/\{2(b-a)\}$. If λ be the dielectric coefficient of the cable with the isotropic dielectric, and $\lambda_{max.}$ be the dielectric coefficient of the inner coating of a graded cable having n layers, the capacities of the cables will be equal if

$$\lambda_{max.} = \lambda(b/a-1)/n\{(b/a)^{1/n}-1\}.$$

If the value of $\lambda_{max.}$ be less than this, the capacity of the graded cable will be the smaller. For example, if there are 4 layers and b/a equals 3, we find that the capacities are equal when $\lambda_{max.} = 1{\cdot}58\,\lambda$. In this case the minimum value of λ in the graded cable is $0{\cdot}69\,\lambda$.

To illustrate how the value of the maximum electric stress diminishes as the number of wrappings is increased we shall work out a few numerical examples.

(i) Two wrappings ($n=2$)—

b/a	2	3	4	5
$R_{max.}/R_{min.}$ graded dielectric	1·414	1·732	2	2·236
$R'_{max.}/R'_{min.}$ isotropic dielectric	2	3	4	5
$R_{max.}/R'_{max.}$	0·828	0·732	0·667	0·618
Per cent. increase of the permissible voltage due to grading	21	37	50	62

(ii) Three wrappings ($n=3$)—

b/a	2	3	4	5
$R_{max.}/R_{min.}$ graded dielectric . .	1·260	1·442	1·587	1·710
$R'_{max.}/R'_{min.}$ isotropic dielectric .	2	3	4	5
$R_{max.}/R'_{max.}$	0·780	0·663	0·587	0·532
Per cent. increase of the permissible voltage due to grading	28	51	70	88

(iii) Four wrappings ($n=4$)—

b/a	2	3	4	5
$R_{max.}/R_{min.}$ graded dielectric . .	1·189	1·316	1·414	1·495
$R'_{max.}/R'_{min.}$ isotropic dielectric .	2	3	4	5
$R_{max.}/R'_{min.}$	0·756	0·632	0·552	0·495
Per cent. increase of the permissible voltage due to grading	32	58	81	102

(iv) Ideal uniformly graded cable ($n=$infinity)—

b/a	2	3	4	5
$R_{max.}/R'_{max.}$	0·693	0·549	0·462	0·402
Per cent. increase of the permissible voltage due to grading	44	82	116	149

We have assumed above that the dielectric strengths of all the insulating wrappings are the same. If, however, the dielectric strengths are known accurately and are not all the same, another solution may be preferable. If R_m be the safe working stress for the mth layer, we have

$$R_m=(V/\lambda_m r_m)/\Sigma(1/\lambda_m)\log_\epsilon(r_{m\ 1}/r_m).$$

Since it is advisable to make the ratio $R_{max.}/R_{min.}$ the same for all layers, the ratio r_{m+1}/r_m will be constant, and as before r_2, r_3, ... r_n will be the n—1 geometrical means between a and b.

The above equation shows that we must have

$$\lambda_1 a R_1=\lambda_2 r_2 R_2= \dots =\lambda_n r_n R_n.$$

Since a, r_2, ... r_n, are in an ascending order of magnitude, $\lambda_1 R_1$, $\lambda_2 R_2$, ... $\lambda_n R_n$, must be in a descending order. We see, therefore, that it is necessary to put the wrappings whose constants are (λ, R) over the wrapping whose constants are (λ', R'), if λR be greater than $\lambda' R'$, even although λ' may be less than λ.

When, however, the main object we have in view is to make, at all costs, the factor of safety of the cable as high as possible, it is, in general, advisable to put the insulating material having the greatest dielectric strength in contact with the core, and, if possible, grade the dielectric by using outer layers having smaller dielectric coefficients.

Grading single core cables for direct pressures

At the moment of switching on the direct pressure, the distribution of the electric stresses depends on the dielectric coefficients of the wrappings, but after a few seconds, when the leakage current attains its steady value, the values of the stresses depend on the insulativities of the various wrappings. If the pressure be always applied gradually at the start we may neglect the dielectric coefficients and grade the cable for the steady pressure taking the insulativities only into account.

Let σ_1, σ_2, ... be the insulativities of the various wrappings and C the leakage current, then (p. 51) the resistance of the mth cylinder to the flow of the current C across it is

$$(\sigma_m/2\pi)\log_\epsilon(r_{m+1}/r_m).$$

Hence we have,

$$C = \frac{V_1 - V_2}{(\sigma_1/2\pi)\log_\epsilon(r_2/a)} = \frac{V_2 - V_3}{(\sigma_2/2\pi)\log_\epsilon(r_3/r_2)}$$

$$= \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot = \frac{V_n - V_{n+1}}{(\sigma_n/2\pi)\log_\epsilon(b/r_n)},$$

and therefore,

$$C = V/\Sigma(\sigma_m/2\pi)\log_\epsilon(r_{m+1}/r_m).$$

Also for the mth layer

$$R_m = -\frac{d\,v}{d\,r}$$

$$= \frac{\sigma_m}{2\pi r_m} C$$

$$= (\sigma_m V / r_m) / \Sigma \{ \sigma_m \log_\epsilon (r_{m+1}/r_m) \}.$$

For reasons stated above, we choose the radii of the boundaries between the wrappings so that they are the $n-1$ geometric means between a and b. Hence, if the factor of safety is to be the same for all the layers, we must have

$$r_m R_m / \sigma_m = \text{constant}.$$

If the dielectric strengths are all equal, this simplifies to

$$r_m / \sigma_m = \text{constant}.$$

Hence, proceeding as in the corresponding problem for alternating pressures, we see that the economies that can be effected by suitably grading the insulativities of the various wrappings are the same in the two cases. It has always to be remembered, however, that the values of the insulativities of insulating materials vary with temperature much more rapidly than their dielectric coefficients. The following table given by A. Campbell (*Proc. Roy. Soc. A.*, vol. lxxviii, p. 196) illustrates the effects of temperature on the physical properties of oven-dried cellulose.

Temperature Centigrade.	Dielectric Coefficient.	Insulativity 10^6 Megohm-cm.
40	6·7	—
25	—	1,600
30	6·8	900
40	7·0	330
50	7·2	125
60	7·3	40
65	—	20
70	7·5	—

The extremely rapid manner in which the insulativity of cellulose varies with the temperature is typical of the behaviour of the other insulating materials used for cables. Hence in connexion with the grading of cables the heating of the dielectric has to be considered. We shall consider this point in the next chapter.

Jona's graded cables

Messrs. Pirelli & Co., of Milan, made experiments on the grading of cables as early as 1898. E. Jona (*Trans. Int. Cong., St. Louis*, vol. ii, p. 550) has constructed single core cables the insulating wrappings of which are arranged so that those nearer the core have greater dielectric coefficients than those more remote. The layers next the core are generally of rubber, and round them are wound layers of paper or jute having smaller dielectric coefficients. The more costly rubber insulation is thus concentrated where its high dielectric coefficient partially relieves the excessive electric stress, and its great dielectric strength enables it to withstand easily this diminished stress. The value generally accepted for the dielectric strength of pure vulcanized para is 15–20 effective kilovolts per millimetre, or 20–30 direct kilovolts per millimetre.

According to Jona, the value of the dielectric coefficient λ of pure vulcanized rubber is 3. We can increase the value of λ without appreciably weakening the dielectric strength by " loading " it with certain materials. The following data, taken from Jona's paper (*l.c. ante*) illustrate that λ can easily be varied throughout wide limits.

	λ
58 per cent. para, 26 per cent. talc, 14 per cent. oxide of zinc, 2 per cent. sulphur	4–4·2
64 per cent. para, 16 per cent. talc, 8 per cent. sulphur, 8 per cent. minium, 4 per cent. oxide of zinc ..	5
55 per cent. para, 22·2 per cent. talc, 22·2 per cent. sulphur	6·1

The following is a description of a Jona graded cable (Fig. 53) which successfully withstood a testing pressure of 150 kilovolts at the Milan Exhibition (1906). The core consists of nineteen strands of copper wire, the diameter

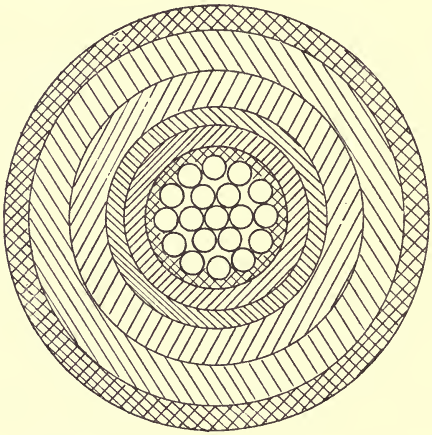

FIG. 53.—Jona's Graded Cable. The stranded core is surrounded with a closely fitting lead tube. There are four insulating layers.

of each of which is 3·3 mm. The cross-section of the copper is therefore 162 mm.². Round this, for reasons explained later, is a close-fitting lead tube, the outer diameter of which is 18 mm. The insulation is built up as follows—

	Thickness in mm.	λ
First layer. Rubber	2·5	6·1
Second layer. Rubber	2·3	4·7
Third layer. Rubber	4·5	4·2
Fourth layer. Impreg. paper	5·2	4·0

The total thickness of the insulation is therefore 14·5 mm. ($b/a = 2·61$), and the cable is lead-covered.

If R, R', R'', and R''', be the maximum electric stresses

on the four layers when the applied pressure is 150 kilovolts, we find by the formula given on page 192, $R=124$, $R'=132$, $R''=123$, and $R'''=97{\cdot}4$ kilovolts per centimetre. If a dielectric of homogeneous substance had been used, the maximum electric stress would have been 174 kilovolts. Hence the grading has reduced the maximum electric stress by about 24 per cent. If air had been the dielectric, a disruptive discharge would have ensued at 23 kilovolts.

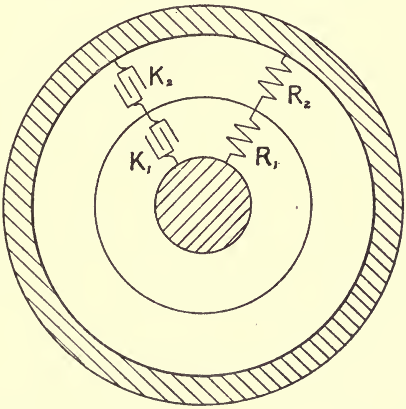

Fig. 54.—Single Core Cable with two homogeneous insulating coverings.

M. O'Gorman has suggested that by suitably "loading" paper insulation we might make the electric stress almost uniform throughout the dielectric. He points out that this can be done in a single core main by arranging that λr is approximately the same at all points.

The effects of leakage currents on the grading of concentric mains

In the discussion given above of the grading of single core mains for alternating pressures, we made the supposition that the leakage currents were negligibly small compared with the capacity currents. We shall now consider how the presence of leakage currents modifies our results. To

simplify the formulae, let us suppose that the dielectric consists of two layers of different isotropic insulating materials at the same temperature throughout. Let σ_1, λ_1 be the insulativity in ohms and the dielectric coefficient of the inner layer next the inner conductor, and let σ_2, λ_2 be the corresponding quantities for the outer layer. Let r be the radius of the cylindrical boundary between the two wrappings (*see* Fig. 54). Then if R_1, R_2 be the resistances per unit length to the flow of leakage electric currents across them, and K_1, K_2 be the capacities in farads per unit length of the cylindrical tubes formed by the wrappings, we have

$$R_1 = \frac{\sigma_1}{2\pi}\log_\epsilon\frac{r}{a}\,; \qquad R_2 = \frac{\sigma_2}{2\pi}\log_\epsilon\frac{b}{r}\,;$$

$$K_1 = \frac{\lambda_1}{2\log_\epsilon(r/a)} \times \frac{1}{9\times 10^{11}}\,; \quad K_2 = \frac{\lambda_2}{2\log_\epsilon(b/r)} \times \frac{1}{9\times 10^{11}}.$$

Now the leakage current i_R across an isotropic dielectric is in phase with the P.D. applied at its boundaries, and the capacity current i_K is 90° in advance of this P.D. If v', v, and v'' denote the instantaneous values of the potential of the inner conductor, the boundary between the two media, and the outer conductor respectively, we have

$$i'_R = \frac{v'-v}{R_1}\,; \qquad i''_R = \frac{v-v''}{R_2}\,;$$

$$i'_K = K_1\frac{d}{dt}(v'-v)\,; \quad i''_K = K_2\frac{d}{dt}(v-v''),$$

where i'_R, i''_R denote the leakage currents, and i'_K, i''_K the capacity currents in the inner and outer wrappings respectively. We also have

$$i'_R + i'_K = i''_R + i''_K = i,$$

since the sum of the leakage and capacity (displacement) currents in each medium must equal the total current flowing across the dielectric.

Let V_1 V_2 denote the effective values of $v'-v$ and of

$v-v''$, and let ϕ_1 and ϕ_2 denote respectively the phase-difference between V_1 and V_2 and the effective value of i. Then if f be the frequency, and $\omega=2\pi f$, we have

$$\left.\begin{aligned} \tan\phi_1 &= \omega K_1 R_1 = f\lambda_1\sigma_1/(18\times 10^{11}), \\ \text{and}\qquad \tan\phi_2 &= \omega K_2 R_2 = f\lambda_2\sigma_2/(18\times 10^{11}). \end{aligned}\right\} \quad \ldots \quad \ldots \quad (A).$$

If, therefore, $\lambda_1\sigma_1=\lambda_2\sigma_2$, then V_1 and V_2 are in phase with one another, and thus

$$V=V_1+V_2,$$

where V is the effective value of the applied P.D. In general, however, $\lambda_1\sigma_1$ is not equal to $\lambda_2\sigma_2$, and therefore V_1+V_2 must be greater than V.

Now by reciprocating the well-known formula (*see* the author's *Alternating Currents* vol. i, p. 166)

$$\frac{A_1}{A}=\frac{\{R_2{}^2+L_2{}^2\omega^2\}^{1/2}}{\{(R_1+R_2)^2+(L_1+L_2)^2\omega^2\}^{1/2}}$$

for the currents in a divided circuit, we get

$$\frac{V_1}{V}=\frac{\{1/R_2{}^2+K_2{}^2\omega^2\}^{1/2}}{\{(1/R_1+1/R_2)^2+(K_1+K_2)^2\omega^2\}^{1/2}} \quad \ldots \quad (B),$$

the formula for the voltages across leaky condensers in series.

By differentiating this expression with respect to ω, it is easy to see that V_1 increases with ω when $\lambda_2\sigma_2$ is greater than $\lambda_1\sigma_1$. In this case, the electric stresses in the medium next the inner conductor increase as the frequency increases, and the stresses in the outer medium diminish.

We see from (B) that, when $\lambda_2\sigma_2$ is greater than $\lambda_1\sigma_1$, V_1/V has its minimum value when ω is zero, that is, with steady pressures, and its maximum value when ω is infinite, that is, with an alternating voltage of very high frequency.

Similarly when $\lambda_2\sigma_2$ is less than $\lambda_1\sigma_1$, V_1/V has its maximum value $R_1/(R_1+R_2)$ with steady voltages, and its minimum value $K_2/(K_1+K_2)$ with alternating voltages of very high frequency.

Numerical example

Let us assume that the radius of the inner conductor is 1 cm. ($a=1$) the radius of the boundary 1·5 cm. ($r=1{\cdot}5$), and the inner radius of the outer conductor 2·25 cm. ($b=2{\cdot}25$). Let us also assume that for the outer jute wrapping, $\sigma_2=10^{12}$, $\lambda_2=2$, and that for the vulcanized rubber inner wrapping, $\sigma_1=10^{16}$, $\lambda_1=4$. If the direct voltage applied to the conductors be 30,000, then, putting $\omega=0$ in (1), we find that

$$V_1=30{,}000, \text{ and } V_2=0, \text{ very approximately.}$$

Thus practically all the electric stress comes on the rubber.

Let us now suppose that an alternating pressure of very high frequency is applied between the conductors. In this case, putting ω equal to infinity in (1), we get

$$\frac{V_1}{V}=\frac{\lambda_2\log_\epsilon(b/r)}{\lambda_1\log_\epsilon(r/a)+\lambda_2\log_\epsilon(b/r)}=\frac{1}{3},$$

and thus, V_1 is 10,000 volts and V_2 is 20,000. Hence, as the frequency increases from 0 to infinity, V_1 diminishes from 30,000 to 10,000 volts, and V_2 increases from 0 to 20,000.

From (A), we see that

$$\tan\phi_1=2\times10^5 f/9, \text{ and } \tan\phi_2=10f/9.$$

Hence, at ordinary frequencies, the error made by assuming that ϕ_1 and ϕ_2 are both 90° is small. If f is greater than 9, V_1/V_2 is nearly equal to 1/2.

In practice, therefore, we see that in the case considered the maximum pressure across the outer layer with alternating pressures may be very much larger than when a direct pressure is applied between the conductors, the value of which equals the maximum value of the alternating pressure between the conductors. On the other hand, the electric stresses on the inner dielectric may be much less with the alternating pressures.

The British standard radial thicknesses for jute and paper dielectrics

The nominal area of the cross-sections of the conductors and the radial thicknesses (b—a) of the dielectric for concentric cables given in the following table are taken from a report issued by the Engineering Standards Committee (E.S.C.) in August, 1904 (p. 8)—

		660 Volts.		11,000 Volts.	
S.	a.	$b-a$.	R	$b-a$.	R_m
sq. in.	in.	in.	K.V. per mm.	in.	K.V. per mm.
0·025	0·089	0·08	0·64	0·35	4·3
0·050	0·126	0·08	0·59	0·35	3·7
0·075	0·155	0·08	0·57	0·35	3·3
0·100	0·178	0·09	0·50	0·36	3·1
0·125	0·199	0·09	0·49	0·36	3·0
0·150	0·219	0·09	0·49	0·36	2·9
0·200	0·253	0·09	0·48	0·36	2·7
0·250	0·282	0·10	0·43	0·37	2·6

In the above table, S represents the cross-sectional area, a the radius of the cylindrical conductor whose cross-sectional area is S, b—a the thickness of the dielectric given by the E.S.C., and R_m the maximum working electric stress when the amplitude factor of the applied alternating pressure is $\sqrt{2}$.

It will be seen that the electric stresses on the dielectric are very different in the high-pressure cable from what they are in the low-pressure cable, and the dielectrics in cables of different sizes are subjected to appreciably different stresses.

In the first five of the high-pressure cables, the dielectric surrounding the high-pressure conductor will begin to be broken down before the disruptive discharge takes place, because in these cables the ratio of b/a is greater than ϵ

(2·718). The specified thicknesses, therefore, are not economical. Take, for instance, the main in which the nominal cross-sectional area of the conductor is 0·025 sq. in. With a solid cylindrical conductor a equals 0·089 in., and b is, therefore, equal to $0{\cdot}35+0{\cdot}089=0{\cdot}439$ in. Thus $b/a=4{\cdot}92$. If we make the inner conductor hollow and $a=0{\cdot}142$ in., $b=0{\cdot}3865$ in., we get the same maximum stress on the dielectric, but its thickness has been reduced by 33 per cent. and the outer radius by 12 per cent. As the armouring, etc., would also be substantially reduced, the cable would be less costly. If we merely kept $b=0{\cdot}439$ in., but increased a to 0·1616, so that $b/a=\epsilon$ nearly, then the carrying capacity of the cable would be nearly quadrupled, the thickness of the dielectric diminished 20 per cent., and the maximum electric stress would have been reduced to 3·8 kilovolts per millimetre.

The fact that the dielectrics of cables are not quite isotropic is sometimes advanced as a reason for making the radius of the inner conductor smaller than the value indicated by theory. This practice, however, is founded on a misapprehension, as the effect of diminishing the radius is to increase the electric stress, and there is no reason why dielectrics of heterogeneous substance should be subjected to greater stresses than those of homogeneous substance. The want of isotropy may possibly be a reason for increasing the diameter of the inner conductor, the thickness of the insulating covering remaining the same.

In designing cables, it has to be remembered that the insulation resistance, the capacity, the electric stresses, and, as we shall see in the next chapter, the thermal conductance have to be considered. For low-tension cables, the insulation resistance must be comparatively high, and hence, it is not possible to use too small a value of b/a.

Similarly, for high-tension cables, although a small value of b/a makes the electric stresses small and increases the thermal conductance yet it makes the capacity large, and the consequent large capacity current may be a serious drawback in practical work.

The effects of stranding on the electric stresses

In order to simplify the formulae for the electric stress, we have assumed that the inner conductor is a smooth cylinder. In practice, the inner conductor is nearly always stranded, and it is necessary therefore to consider the effect of the stranding. Owing to the greater curvature of the surface of the strands, we can see, from first principles, that the effect will be to increase the maximum stress. Jona found experimentally that the brush discharges from solid wires and stranded or braided wires having the same external size begin at practically the same voltages. Hence we may infer that the stranding of the conductor does not much affect the dielectric strength of the cable. It is important, however, to be able to calculate the stress exactly, and this can be done by means of a formula due to Professor Levi-Civita (*vide* Jona *l.c. ante*). The formula is given in terms of Gauss's hypergeometric series, but Jona has computed these series for useful values of the variables, so that approximate solutions can be readily obtained. The results show that the effect of the stranding is generally to increase the maximum stress on the inner dielectric by about 20 per cent. It is worth while, therefore, to prevent this increase in the stress on the inner wrapping by making the surface of the conductor smooth. This can be done by covering, as Jona does, the inner conductor with a thin lead tube. For extra high-pressure cables the gain in the strength is well worth the slight increase in the cost of the cable.

Conclusions We may sum up the results arrived at in this and the preceding chapter as follows.

(1) When part of the dielectric under stress breaks down, a disruptive discharge ensues only when the effect of this partial breakdown is to increase the electric stress on the remaining portion.

(2) The dielectric strength of air under given conditions can be found accurately by finding the disruptive voltages between spherical electrodes at distances greater than 0·5 of a centimetre apart. Under normal conditions it is about 3·8 kilovolts per millimetre.

(3) The dielectric strength of other gases can be found in a similar way experimentally by the help of the tables given on p. 176. The dielectric strengths of the monatomic gases helium and neon are small.

(4) The dielectric strength of oils can be found by noticing the disruptive voltages between spherical electrodes immersed in them, provided that the distance apart is greater than 0·3 of a centimetre. An excellent way of drying oils is by letting heated air bubble through them.

(5) In finding the dielectric strength of solids it is advisable, when possible, to embed the spherical electrodes in the material under test.

(6) High-pressure concentric cables having an isotropic dielectric, for a maximum working pressure V should be constructed so that—

$$b = a\epsilon^{l/a},$$

where V/l is the maximum permissible working stress to which the dielectric may be subjected, b is the inner radius of the outer conductor, and a is the outer radius of the inner conductor. The smallest permissible value of a is l. When the core is stranded it should be encased in a thin lead tube.

(7) With a composite dielectric subjected to alternating pressure, the P.D.s across the layers are usually out of phase with one another. It is only in a limited number of cases, however, that the increase of the stress due to this cause has to be considered, as the leakage currents are usually negligibly small in comparison with the capacity currents.

(8) The effects of alternating and direct pressures in producing stresses in the dielectric are sometimes quite different.

(9) High-pressure cables for alternating or direct current circuits should be graded so as to make the maximum electric stress on the dielectric as small as possible, and stranded conductors should be encased in thin lead tubes.

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THE HEATING OF CABLES

CHAPTER X

The Heating of Cables

The heating of cables—Temperature gradient in a concentric main—Numerical example—Effects of heat on the dielectric—Effect of the temperature gradient on the electric stress—The thermal conductance of a single core main—Numerical example—The temperature in the substance of conductors carrying uniform currents—The thermal conductance of a concentric main—The thermal conductance of polycore cables—The heating of bare conductors—References.

The heating of cables

ALTHOUGH the rise of temperature in underground cables is a problem of considerable practical importance, yet very little information on this subject is available. It is usual to consider that a cable of given dimensions is carrying its full load when the current density in the core has a given value. As a rule the effect of the thermal conductivity of the insulating wrappings is not taken into account. The value of this physical constant, however, determines the difference of temperature between the core and the sheath, and hence the temperature, and consequently the electrical conductivity of the copper must be considerably affected by the value of the thermal conductivity of the dielectric. For high-pressure cables also the thermal gradient in the dielectric affects, in some cases very seriously, the dielectric strength. We shall therefore briefly consider the laws governing the flow of heat across the dielectric of cables.

Temperature gradient in a concentric main

When a concentric main is carrying a current, the temperature of the dielectric is not uniform owing to the heat generated in the inner conductor. If the dielectric is isotropic, the temperature at any point after the flow of heat has become steady can be readily written down, if we assume that the thermal conductivity k of the dielectric remains approximately constant over the range of working temperatures.

If θ be the temperature at all points at a distance r from the axis of the main, we have, since the heat entering per second an elementary cylinder of the dielectric, coaxial with the main, must equal the heat leaving it,

$$\frac{d}{dr}\left(2\pi r k \frac{d\theta}{dr}\right) = 0,$$

neglecting the flow of heat near the ends parallel to the length.

Hence
$$\frac{d\theta}{dr} = -\frac{A}{r},$$

where A is a constant. We have, therefore,

$$\theta = \theta_2 + A \log_\epsilon(b/r),$$

where θ_2 is the temperature of the outer conductor, the inner radius of which is b.

Let us suppose that the inner conductor, supposed of copper, is solid and of radius a, and that i is the current density in it. Then, if ρ be the volume resistivity of the copper in ohms, we have

$$-4{\cdot}2 \times 2\pi a k \left(\frac{d\theta}{dr}\right)_{r=a} = (\pi a^2 i)^2 \frac{\rho}{\pi a^2},$$

and thus,
$$A = a^2 i^2 \rho/(8{\cdot}4 k).$$

Hence
$$\theta = \theta_2 + (a^2 i^2 \rho/8{\cdot}4\, k) \log_\epsilon(b/r),$$

and
$$\theta_1 - \theta_2 = (a^2 i^2 \rho/8{\cdot}4\, k) \log_\epsilon(b/a),$$

where θ_1 is the temperature of the surface of the inner conductor.

We have assumed above that the thermal conductivity k of the dielectric does not vary appreciably with the temperatures likely to occur in practice. C. H. Lees (*Trans. Roy. Soc.*, p. 433, vol. 204A, 1905) has proved that this assumption is permissible for paraffin wax, glycerine, and various other insulating materials. There appears to be a slight tendency, however, towards lower conductivity as the temperature increases. G. F. C. Searle (*Proc. Camb. Phil. Soc.*, xiv, 2, p. 189, 1907) has devised an exceedingly simple method of determining the thermal conductivity of rubber, the value of which he finds to equal 0·0004 nearly.

Numerical example To illustrate the values of $\theta_1-\theta_2$ likely to occur in practice, let us suppose that $b=1{\cdot}649$ cm., and $a=1$ cm. Let us also suppose that the current density i is 150 amperes per sq. cm., that $\rho=1{\cdot}8\times10^{-6}$ and that $k=0{\cdot}0006$. The author has no trustworthy data with reference to the conductivities of the dielectrics used in actual cables, and so he takes the value of k for paraffin wax, which has been found accurately by Lees (*l.c. ante*). Substituting in the formula, we get

$$\theta_1-\theta_2=\frac{(150)^2\times1{\cdot}8\times10^{-6}}{8{\cdot}4\times0{\cdot}0006}\times\tfrac{1}{2}$$

$$=4^\circ\text{ C. nearly.}$$

It is easy to see from the formula for $\theta_1-\theta_2$, that for a given value of b and for a given current density, the difference of temperature between the inner and outer conductors is a maximum when

$$a=b/\sqrt{\epsilon}=b/1{\cdot}649=0{\cdot}6065\,b,$$

which is the case we considered. We see, therefore, that the difference of temperature between the inner and outer conductors is probably not greater than 10 deg. in the most unfavourable circumstances.

Effects of heat on the dielectric

It is known that the dielectric coefficient and the electric insulativity of an insulating material vary rapidly with the temperature. Jona (p. 207) mentions a case where a rise of temperature of 20° C. made the insulation resistance of a paper insulated cable fall to one-thirtieth of its original value, and even more striking instances could be given. The table, also, quoted in the last chapter (p. 196) for oven-dried cellulose shows that the dielectric coefficient varies from 6·7 to 7·5 as the temperature rises from 20 to 70 degrees Centigrade. In most cables it is noticed that the capacity current increases and the insulation resistance diminishes as the temperature rises. The rise of the capacity current is probably generally due to the increase in the value of the dielectric coefficient. We see therefore that in practice, since the temperature of the dielectric is not uniform, both ρ and λ vary, even when the insulating covering is made of homogeneous material. We shall now consider what effect this has on the electric stresses in the insulating material.

Effect of the temperature gradient on the electric stress

Let us suppose that a steady pressure E is applied across the inner and outer conductors of a concentric main having an isotropic dielectric. The momentary stresses set up initially are the same as if the resistivity were infinite. Now imagine that the dielectric is split up into an infinite number of concentric cylindrical tubes, the material of each tube being at the same temperature. Since these tubes form condensers in series between the conductors, the quantity of electricity per unit length in each condenser will be the same, and thus

$$\lambda \frac{2\pi r}{4\pi dr} dv = \text{constant},$$

where λ is the value of the dielectric coefficient at a distance

r from the axis, and v is the potential at the same distance.

Hence

$$-\frac{dv}{dr}=\frac{A}{\lambda r},$$

where A is a constant. Now λ diminishes as the temperature diminishes, it therefore diminishes as r increases. We see, therefore, that the effect of λ varying with the temperature is to make the electric stress on the dielectric more uniform.

If we assume that λ varies with temperature according to the linear law, we may write

$$\begin{aligned}\lambda &= \lambda_o\{1+a(\theta-\theta_2)\}\\ &= \lambda_o\{1+B\log_\epsilon(b/r)\},\end{aligned}$$

where $B=a\,a^2\,i^2\rho\,/8{\cdot}4k$. It readily follows that the electric stress is a minimum where

$$r=b\epsilon^{\,1-B)/B}.$$

In practice, B is very small compared with unity, and hence the electric stress diminishes as we pass from the inner to the outer conductor.

Let us now suppose that the direct pressure E has been applied sufficiently long to make the electric stresses and the leakage currents assume their steady values. In this case, by Ohm's law,

$$dv\Big/\left\{\sigma\frac{dr}{2\pi r}\right\}=\text{constant},$$

where σ is the insulativity, in ohms, of the dielectric.

Hence $$\frac{r}{\sigma}\frac{dv}{dr}=-A',$$

or $$-\frac{dv}{dr}=\frac{\sigma A'}{r},$$

where A' is a constant.

Now σ increases as the temperature diminishes and therefore as r increases. The variation of σ, therefore, due

to a slight temperature gradient in the dielectric tends again to make the stress more uniform. But if there be a drop of 10° C. between the inner and outer conductors the electric stresses on the outer layers of the dielectric when the cable is loaded may be much greater than on the inner layers.

The temperature in the substance of conductors carrying uniform currents

The variation of temperature over the cross-section of a conductor, once the flow of heat has attained its steady state, is very small. Let us consider, for instance, a cylindrical copper conductor carrying a constant current the density of which is i, so that the total current is $i.\pi R^2$. Then when the flow of heat has become steady, the heat per unit length flowing across the surface of an elementary concentric cylinder of radius r must equal the heat being generated per unit length by the current flowing inside this elementary cylinder. Thus we have

$$k.2\pi r(d\theta/dr) = -(1/4{\cdot}2)i^2(\pi r^2)^2\rho/\pi r^2$$
$$= -\rho\pi i^2 r^2/4{\cdot}2.$$

Therefore $$d\theta/dr = -(\rho i^2/8{\cdot}4k)r$$

and thus $$\theta = \theta_o + (\rho i^2/16{\cdot}8k)(R^2 - r^2).$$

If θ_1 be the maximum temperature which will be along the axis of the conductor, we have

$$\theta_1 - \theta_0 = \rho i^2 R^2/16{\cdot}8\ k.$$

As an example let us take the case of a cylindrical copper main, the diameter of which is 2 cms., so that $R=1$. We shall suppose that $\rho = 1{\cdot}68 \times 10^{-6}$ ohms, $i=100$ amperes and $k=0{\cdot}96$ C.G.S. units. In this case the maximum difference $\theta_1 - \theta_0$ between any two points on the cross-section of the main is given by

$$\theta_1 - \theta_0 = 1{\cdot}68 \times 10^{-6} \times 10^4/(16{\cdot}8 \times 0{\cdot}96)$$
$$= 0{\cdot}001^\circ\ \mathrm{C}.$$

Hence no appreciable error is made in this case by the

assumption that the temperature is uniform over the cross-section of the core. Even if we suppose that the resistivity of the metal is ten times that of copper and that the current density is 1,000 amperes per square centimetre, the maximum difference of temperature would only be 1° C.

The thermal conductance of a single-core main

The thermal conductance of the dielectric of a single-core cable is the ratio of the flow of heat per second across the dielectric, in gramme-Centigrade units, to the difference in temperature (Cent.) between the outer boundary of the core and the inner boundary of the lead sheath, when the flow of heat has attained its steady state. As the thermal conductivities of insulating materials are very small compared with those of metals, we can assume, without appreciable error, that the metals are at uniform temperatures. If a and b be the inner and outer radii of the dielectric (supposed isotropic) we have (p. 212)

$$\theta = \theta_2 + (\theta_1 - \theta_2)\{\log_\epsilon (b/r)/\log_\epsilon(b/a)\},$$

and

$$Q = -k.2\pi r(d\theta/dr)$$
$$= 2\pi k \frac{\theta_1 - \theta_2}{\log_\epsilon(b/a)},$$

where θ is the temperature of the dielectric at points distant r from the axis of the cable, θ_1 and θ_2 the temperatures of the core and sheath respectively, and Q is the thermal flow per unit length in calories per second. Hence the thermal conductance per unit length for a single-core cable is $2\pi k/\log_\epsilon(b/a)$. It is therefore equal to $4\pi k/\lambda$ times the corresponding electrostatic capacity. If r_1 be the electric resistance of the conducting core per unit length and C the current flowing in it, then, when the thermal flow attains its steady value, $Q = C^2 r_1/4{\cdot}18$, and thus $\theta_1 - \theta_2$ can be readily found if k be known.

Numerical examples

Let us assume that the diameter of the conductor of a single-core main is 1 centimetre, and that a current of 100 amperes is flowing in it. Let us also assume that $b/a=2{\cdot}718$, and that k is $0{\cdot}0004$. In this case, r_1 will be $0{\cdot}2\times10^{-5}$ ohms approximately, and thus $Q=C^2r_1/4{\cdot}18=0{\cdot}02/4{\cdot}18=0{\cdot}004785$.

Hence since

$$\theta_1-\theta_2=(Q/2\pi k)\log_\epsilon(b/a),$$

we readily find that

$$\theta_1-\theta_2=1{\cdot}9^\circ \text{ C. nearly.}$$

If we suppose that k is greater than $0{\cdot}0004$ the difference of the temperatures will be less than this.

The thermal conductance of a concentric main

We shall now consider a concentric main. Let the conductivity of the isotropic dielectrics between the two conductors and between the outer conductor and the lead sheath be k_1 and k_2 respectively. Then if θ_1, θ_2, and θ_3 be the temperatures of the two conductors and the sheath, we have

$$Q/2=2\pi k_1(\theta_1-\theta_2)/\log_\epsilon(b/a),$$

and

$$Q=2\pi k_2(\theta_2-\theta_3)/\log_\epsilon(d/c),$$

where $Q/2$ is the heat generated per unit length in each conductor per second, and c and d are the radii of the outer dielectric. Hence

$$\frac{Q}{\theta_1-\theta_3}=\frac{2\pi}{(1/k_2)\log_\epsilon(d/c)+(1/2k_1)\log_\epsilon(b/a)}.$$

If we can write $b=c$, and $k_1=k_2$, without appreciable error, we get $2\pi k_1/\log_\epsilon(d/\sqrt{ab})$ for the thermal conductance.

The thermal conductance of polycore cables

The problem of the thermal conductance of a polycore cable is much more difficult than that of the single-core cable. When the dielectric is isotropic, formulae for the electrostatic capacity between the cores in parallel and the sheath are given in

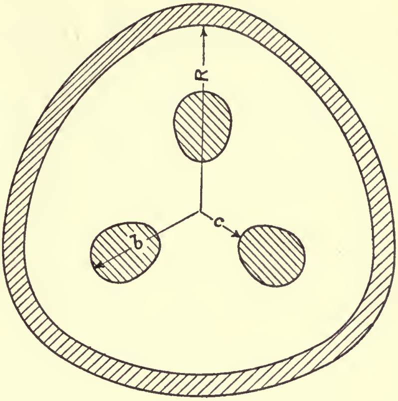

Fig. 55.—The section of a Three-core Main of which the thermal conductance can be accurately calculated.

Russell's *Alternating Currents*, vol. i, chap. v. The corresponding thermal conductances can be at once deduced from these formulae by multiplying the capacities by $4\pi k/\lambda$. In a three-core cable, for instance, of a certain "clover leaf" pattern (Fig. 55), if Q be the heat generated in the three cores, per unit length, we have

$$\frac{Q}{4\pi}=\frac{k(\theta_1-\theta_2)}{(2/3)\log_\epsilon[\{2R^3-(b^3+c^3)\}/(b^3-c^3)]},$$

exactly, where R is the maximum inner radius of the lead sheath and b and c are the maximum and minimum distances of points on the cores from the axis of the cable.

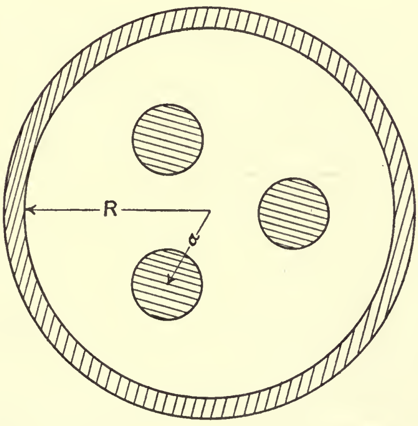

Fig. 56.—Section of another ideal Three core Main, of which the thermal conductance can be calculated approximately.

By using Kelvin's method of images, it may be shown that if the centres of the n cores are symmetrically situated on a circle of radius a (Fig. 56), and if the cross-sections of the cores are small circles of radius r, we have,

$$\frac{Q}{4\pi}=\frac{k(\theta_1-\theta_2)}{(2/n)\log_\epsilon\{(R^{2n}-a^{2n})/nR^n a^{n-1}r\}},$$

approximately, where R is the inner radius of the lead sheath. When $n=3$ this formula agrees with that given above, provided that r/a and $(a/R)^3$ can be neglected compared with unity.

It is to be noticed that all the logarithms given in this chapter and the preceding two chapters are Neperian. Their values are best found directly from Bottomley's Tables. They may also be found with an accuracy sufficient for practical purposes by multiplying the corresponding ordinary logarithms by 2·3. In many cases they may be computed easily, without tables, by means of the formula

$$\log_\epsilon(b/a)=2x+2x^3/3+2x^5/5+\dots,$$

where

$$x=\{(b/a)-1\}\{(b/a)+1\}.$$

For instance

$$\log_\epsilon 1{\cdot}5=0{\cdot}4+0{\cdot}016/3+\dots$$
$$=0{\cdot}405.$$

The heating of bare conductors

We shall conclude this chapter by discussing the variation of the temperature of a bare cylindrical conductor, through which electrical currents are passing, when suspended horizontally. We have seen that no appreciable error is made by the assumption that the temperature of the conductor is uniform over its cross-section. We shall make the further assumption that the rate at which the wire radiates heat is proportional to its surface and to the difference of temperature between the wire and its surroundings. Hence for rises of temperature greater than about 50° C. our results are only rough approximations. The temperatures of the conductor also will not be the same during a gale or when it is raining as they would on a calm dry day. The formulae, however,

indicate how the temperatures vary with the dimensions of the conductor.

Let us first suppose that when the switch is closed a current C flows in the conductor, and that this current is maintained constant. Let ρ be the volume resistivity, D the density, h the emissivity for heat of the surface, c the specific heat, l the length, and r the radius of the conductor. If θ be the temperature of the wire, at the time t after the switch has been closed, we have

$$0{\cdot}239C^2(\rho l/\pi r^2)=h.2\pi rl.\theta+d(D\pi r^2l.c.\theta)/dt,$$

since the electric power, expressed in calories per second, being expended in the wire equals the rate at which heat is being radiated from the surface together with the rate at which it is being stored in the substance of the conductor. Assuming for the present that ρ is constant, we see that the solution of the above equation is

$$\theta=\theta_1(1-\epsilon^{-mt}),$$

where $\theta_1=0{\cdot}239C^2\rho/(2h\pi^2r^3)$ and $m=2h/Drc$.
The final temperature of the wire is θ_1 which varies directly as $C^2\rho$ and inversely as hr^3.

Let us next suppose that the voltage V at the terminals of the wire is maintained constant. The equation now becomes

$$0{\cdot}239V^2(\pi r^2/\rho l)=h.2\pi rl.\theta+D\pi r^2lc(d\theta/dt),$$

and thus,

$$\theta=\theta_1'(1-\epsilon^{-mt}),$$

where,

$$\theta_1'=0{\cdot}239(V^2/\rho)(r/2hl^2).$$

Hence the final temperature varies directly as (V^2/ρ) and r, and inversely as hl^2. The greater the value of m the more rapidly does the temperature rise to its steady value. The greater the value of the emissivity, therefore, and the

smaller the value of the density, the radius, and the specific heat, the more rapid will be the rise of temperature.

Let us now suppose that ρ varies with the temperature. In this case we may assume that $\rho=\rho_0(1+a\theta)$ where a is a constant. When the current is constant the temperature equation now becomes

$$0{\cdot}239C^2(\rho_o l/\pi r^2)=(2\pi rhl-0{\cdot}239C^2\rho_o al/\pi r^2)\theta+D\pi r^2lc(d\theta/dt),$$

and hence,

$$\theta=\theta_1(1-\epsilon^{-mt}),$$

where $$\theta_1=0{\cdot}239C^2\rho_o/(2h\pi^2r^3-0{\cdot}239C^2\rho_o a),$$

and $$m=(2h-0{\cdot}239C^2\rho_o a/\pi^2r^3)/Drc.$$

Again when the applied voltage is constant, noticing that $1/\rho=1/\rho_o-a\theta/\rho_o$ approximately, we have

$$0{\cdot}239V^2(\pi r^2/\rho_o l)=\{2\pi rhl+0{\cdot}239V^2a(\pi r^2/\rho_o l)\}\theta + D\pi r^2lc(d\theta/dt),$$

and thus $$\theta'=0{\cdot}239V^2r/(2hl^2\rho_o+0{\cdot}239V^2ar),$$

and $$m'=\{2h+0{\cdot}239V^2a(r/\rho_o l^2)\}/Drc,$$

and the temperature at any instant is given by

$$\theta=\theta'(1-\epsilon^{-m't}).$$

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ELECTRICAL SAFETY VALVES

CHAPTER XI

Electrical Safety Valves

Electrical safety valves—Intermittent safety valves—Siemens and Halske horn arrester—The Seibt safety valve—Multiple gap lightning arresters—Pressure safety valves on a 3-phase line—Continuous arresters—Electrolytic arresters—References.

Electrical safety valves

In practical working, the machines, apparatus, and cables, used for high pressure networks are often subjected to abnormal stresses owing to a sudden rise of pressure. This rise of pressure may be due to atmospheric electricity, to resonance due to a harmonic of the applied E.M.F. wave having the same period as a free period of vibration of the system, or to electromechanical resonance between the prime mover and the oscillating electrical energy. It may also be due to resonance of the high frequency oscillations often set up when an arc occurs at a short circuit. Hence excessive rises of pressure can occur on a direct current network when an arc giving rise to Duddell currents exists on any part of the circuit. The impulsive rush of electricity also, at these high frequencies, often causes momentary disruptive discharges, making pinhole marks, through the dielectric at places where the rush meets with inductive resistance. Between the first two turns, for instance, of a coil in the circuit or at a sudden bend of the conductor. The most frequent cause of breakdown is due to the oscillating arc,

but the most disastrous occur when the period of one of the free oscillations of a network has the same value as a harmonic of low order of the impressed oscillations. In this case the amplitude of the pressure between two points may reach enormous values, and considerable power may be expended where a breakdown occurs.

To prevent the breakdown of the cables from these causes, or of the insulation of the armatures of the dynamos and other appliances in the network, electrical "safety valves" must be provided. These devices may be classed under two main heads, (1) intermittent safety valves, that is, devices which only act when the pressure exceeds a certain critical value, and (2) continuous safety valves which are always in operation. The first type acts by providing a safety path for the oscillating charge when its value gets excessive. The second type acts by conduction. It prevents the accumulation of an excessive charge by allowing it to leak away by a path of small resistance which is always in circuit.

Intermittent safety valves

In the usual types of intermittent safety valves, the two electrodes are separated by a suitable dielectric which is usually either air or oil. One electrode is connected with one main, and the other with another main of different polarity. When the difference of the pressure between the two exceeds a certain value the dielectric breaks down, and the ensuing arc having a very small resistance, the pressure between the mains to which the device is joined cannot attain a high value. The ensuing arc is broken in several ways, some of which are described below.

If one of the electrodes is connected with a main, and the other with an earth plate, the device is generally called a lightning arrester. A device used to limit the pressure

can also be used as a lightning arrester. The breadth of the gap between the electrodes, and the design of certain auxiliary apparatus, however, is generally different in the two cases.

Siemens and Halske horn arrester

The Siemens and Halske horn arrester (Fig. 57) is perhaps the one most extensively used on power circuits. When the pressure between the main *M* and the earth *E* exceeds a certain value, an arc ensues between the narrowest portion of the gap between the two horns. It then rapidly travels upwards until its length gets too great for the voltage at its terminals when it automatically ruptures. Although the rupture of the arc is accelerated by the convection currents of air yet it is partly also an electromagnetic phenomenon. If we invert the arrester, for instance, the arc travels downwards if it is started below the narrowest point (*see* A. Moore, *Elect. Engineer*, vol. 34., p. 520, 1904).

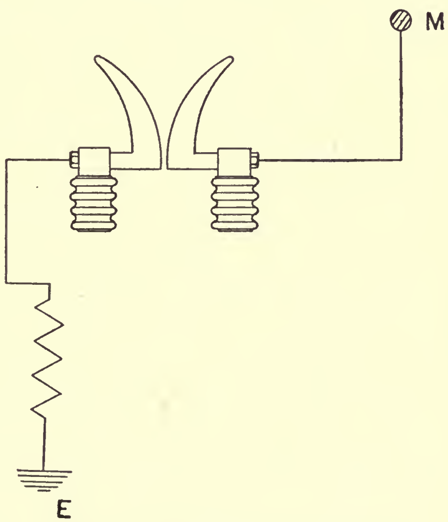

FIG. 57.—Horn Lightning Arrester.

The Oerlikon Company use a horn arrester with a non-inductive resistance in series with it. This is formed of nickeline wire immersed in oil and bent so as to be practically non-inductive. The use of this resistance is to obviate the dangers arising from possible high frequency oscillations being set up at the arc, and also from oscillations being set

up by a sudden rush of current through a path of small resistance. The Allgemeine Elektricitäts Gesellschaft have employed for the series resistances 150 watt, 150 volt glow lamps connected in series, a sufficient number being employed to prevent any of them burning out and breaking the circuit.

To prevent oscillations, and to provide a path of small resistance for the discharge, it is important to have the inductance of these circuits as low as possible. In practice, it is difficult to make the inductance of a wire circuit small enough, and hence carbon cylinders, water resistances, and even wet sand are sometimes employed. The function of these resistances is to get rid of most of the destructive energy contained in violent atmospheric discharges.

W. H. Patchell (*Journ. Inst. El. Engin.*, vol. 36, p. 97) has found that a spark-gap, with one electrode of copper and the other of carbon, in a glass enclosure is very effective as a safety-valve. The travelling of the spark upwards in the gap between the horns is accelerated by the chimney action of the enclosure, and many tests have proved that this type can be calibrated more accurately, and adjusted within narrower limits, than the ordinary open horn type with copper electrodes. A liquid resistance is used consisting of a solution of glycerine and water contained in earthenware vessels, but it has not proved altogether satisfactory as the values of the resistances are liable to change. The width of spark-gap employed is 4·5 mms. for $10{,}000/\sqrt{3}$, that is, about 5,800 volt working. A spark will jump the gap and start the arc at a pressure of 12,000 effective volts when the horns are clean and the atmosphere is normal.

When these safety-valves were first installed in the City of London Works of the Charing Cross Company, the irregular times at which they acted attracted attention.

To discover the cause a detector was extemporized for experimental use. The primary of a small transformer was inserted in the earth wire of the spark-gap resistance. The secondary acted a relay which rang a bell and thus attracted the attention of the engineer. It was found that irregularities in starting a machine, although not sufficient to prevent easy synchronizing and switching in, were a frequent cause of spark discharges. An interruption in the supply due to a faulty insulator nearly always caused a discharge. It is probable, therefore, that the rises of pressure in the

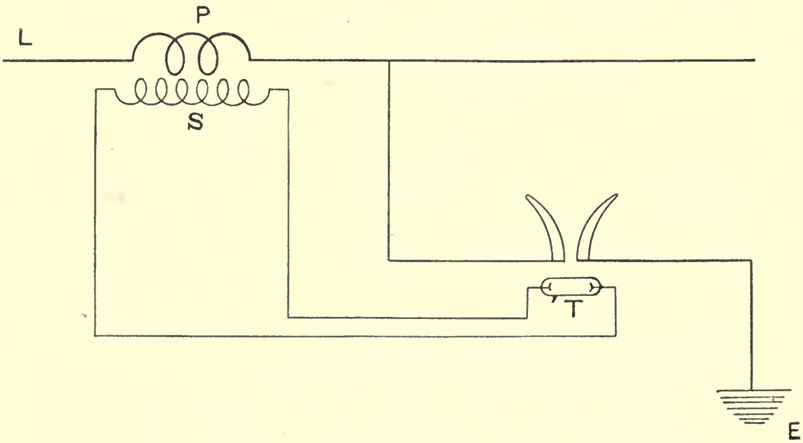

Fig. 58.—Seibt Lightning Arrester.

network were due to the superposition of the free oscillations set up by the disturbance on the normal oscillations. The indicating device has proved so useful that it has been adopted permanently. A time recorder could also easily be actuated by the transformer, the inductance of the primary of which can be made very small.

The Seibt safety valve

A drawback to the use of the horn arrester is the large factor of safety that has to be allowed in order to avoid unnecessary sparking.

Hence it might easily happen that the excessive electric stress did serious harm before the valve acted. This difficulty is neatly surmounted in the Seibt safety valve (Fig. 58), by utilizing the well-known effect of ultra-violet radiation in lowering the value of the disruptive voltage required by an air gap. The primary P of a small transformer is put in series with the line L, and the secondary S consists of many turns of fine wire. A vacuum tube T placed between the secondary terminals glows when high frequency oscillations are set up in the main, and the dielectric strength of the air in the spark-gap safety valve being lowered by the radiations from the vacuum tube, a disruptive charge takes place to the earth E, and thus the pressure is prevented from becoming excessive.

Multiple gap lightning arresters

The multiple gap lightning arrester (Fig. 59) is a type of arrester frequently used in power transmission circuits in America. Between the line M and the earth E there is a series of insulated con-

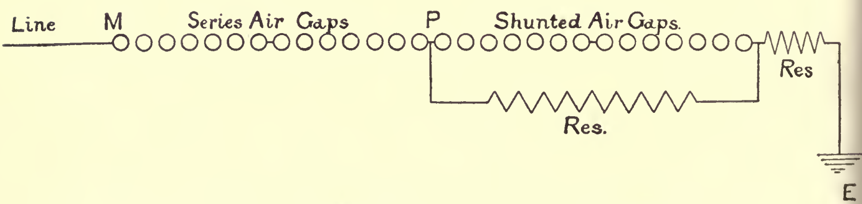

FIG. 59.—Multiple Gap Lightning Arrester.

ductors, having small air gaps between them, and there is a resistance in series with the last of the conductors. In order to prevent an excessive rush of current from the line when the device acts, it is necessary to make this resistance of appreciable magnitude which is an objectionable feature as it lowers the efficiency of the device. To obviate this difficulty another resistance is placed as a shunt to half

the conductors (Fig. 59), and it is consequently in series with the first resistance. Since P is at earth potential initially, a discharge ensues when the voltage is sufficiently high to break down the series air gaps from M to P. The impulsive rush of current which ensues breaks down the shunted air gaps and gets to earth by the series resistance. When the impulsive rush is over the arcs across the shunted air gaps go out, being shunted by a resistance. Both resistances in series now carry the current through the arcs between the series air gaps, and hence, when the voltage becomes normal, these arcs go out and the leakage of current from the main is stopped. It will be seen that the use of a resistance shunting some of the air gaps makes the device much more effective.

Pressure safety valves on a three phase line

The *Société d'énergie électrique de Grenoble et Voiron* have found the following arrangement of lightning arresters and pressure safety valves very satisfactory in practical working for an overhead 15,000 volt network, extending over 60 kilometres, in a district subject to severe thunderstorms. In Fig. 60, three horn lightning arresters, outside the power station S, one connected with each main, are represented at A. The minimum width of the air gap is 13 mms., and each earth circuit is composed of damp sand the resistance of which is about 8,000 ohms. No choking coils are placed between the arresters and the mains, but the inductance of the connecting wires is increased by bending them at sharp angles. The safety valve P, limiting the pressure between the mains, is in the station itself, and consists of three Siemens horn arresters, connected in mesh, and having resistances of 15,000 ohms in circuit with them.

Originally lightning arresters were placed at distances

of 2·5 kilometres apart all along the line. These are now replaced by two lightning arresters the positions of which have been carefully chosen. These have been found quite sufficient to protect the line insulators from damage during

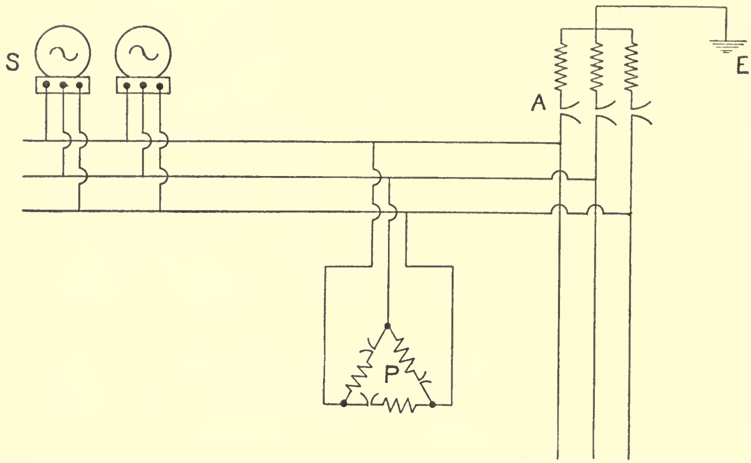

FIG. 60.—Lightning Arrester *A* and device *P* for limiting the pressure.

thunderstorms. Interruptions to the working of the line due to sudden falls of the potential difference and short circuits, which were formerly often caused by the irregular action of the earlier types of arrester used, now practically never occur.

At the transformer substations either horn or multiple-gap lightning arresters are used. No other special safety valve apparatus is employed to limit the rise of pressure between the mains at the substations.

Continuous arresters

In this type of arrester resistances through which a current is continually leaking are interpolated between the lines and earth. Hence it is essential that during normal working the losses due to these arresters should not be large. A jet of water is generally employed

in continuous arresters to carry the leakage current, as a comparatively large amount of energy can be got rid of in this way without using costly resistances, and there is no risk from over heating.

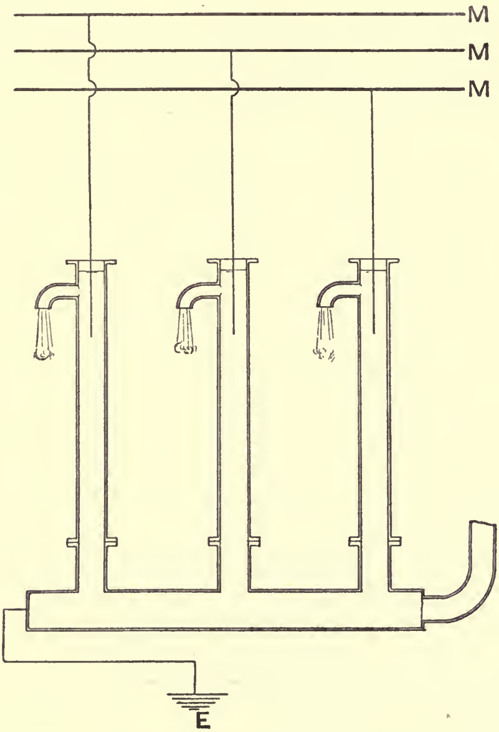

Fig. 61.—Continuous Arrester.

The *Société hydro-électrique de Vizille* distribute power, by a 3-phase overhead network 40 kilometres long, at a pressure of 10,000 volts. As this pressure is obtained directly from the terminals of the machines, special precautions have to be taken to protect the armatures of the machines from damage by sparks due to atmospheric dis-

charges causing short circuits or damaging the insulation. When the network was first installed only rough types of arresters were used, and as thunderstorms are frequent and severe in this district the 3-phase machines were often damaged.

At the power station a continuous arrester (Fig. 61) is employed and, in addition, choking coils are placed in series with the mains to hinder impulsive rushes of electricity from getting to the terminals of the machines, and thus into the armature. At the transformer substations, horn arresters with resistances in series with the earth connexion are used. The resistances are simply stoneware tubes, 80 centimetres long, filled with water.

The continuous arrester (Fig. 61) consists of three stoneware tubes each of which is 2·5 metres long and 15 centimetres in diameter. They are fixed in an iron pipe 5 metres long and 20 centimetres in diameter which is in connexion with a good earth. A current of water is continually flowing up the stoneware tubes and escaping from the waste pipes near the top. Each of the three mains is in direct contact with the water through a wire which dips into it to a depth of a few centimetres. The current in each wire during normal working is about 0·3 of an ampere and thus the power expended in this device is $\sqrt{3} \times 10000 \times 0{\cdot}3$ watts, that is, about 5 kilowatts.

This continuous pressure arrester has been in use for some years, and no accidents to the alternators, due to atmospheric electricity, which were formerly frequent, now occur.

Another type of continuous arrester due to *La Société d'Applications Industrielles* is shown in Fig. 62. The mains are connected with earth by means of vertical jets of water which play against metallic cups, each cup being in direct connexion with a main through a wire. Particular care

has to be taken that the pipe bringing the water has a good earth connexion.

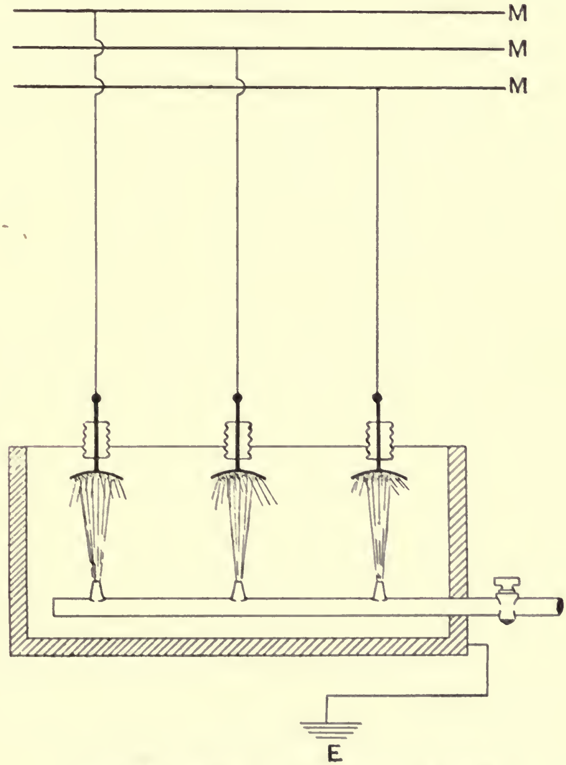

FIG. 62.—Water Jet Continuous Arrester.

Electrolytic arresters

Various types of electrolytic cell possess unilateral conductivity, that is, they allow the electric current to pass through them much more readily when it flows in one direction than when it flows in the other. An electrolytic cell may be made by immersing one electrode of aluminium, and one of some other conducting substance, in an electrolyte. Dilute sulphuric acid, bichromate solution, ammonium phosphate

solution, etc., are suitable electrolytes. If the aluminium electrode be at a higher potential than the other, and if the P.D. between them be less than a certain critical value, very little current will flow through the cell. This is owing to the formation of a thin film of high-resistance material round the aluminium electrode. K. Norden (*Electrician*, vol. xlviii, p. 107) has found that this film consists of normal aluminium hydroxide [$Al_2(OH)_6$]. If the direction of the applied voltage be reversed, the film dissolves rapidly, and the effective resistance of the cell is very considerably reduced.

When the electrolytic cell is placed in an alternating current circuit, and the maximum value of the applied P.D. is less than the critical voltage for the cell, the current flowing through it during the half period when the aluminium electrode is at the lower potential will be much greater than during the other half period and hence we get what is practically a pulsating direct current. This is the principle utilized in the Pollak electrolytic rectifier, and in the Nodon valve. Both of these rectifiers can be employed, for example, for charging direct current accumulators from the alternating current mains.

If we gradually raise the direct voltage applied to the terminals of an electrolytic cell, the aluminium electrode of which is connected with the positive main, then, when the critical voltage is passed the current increases and the resistance of the cell diminishes very rapidly. It is this action of the cell that makes it valuable as an electrical safety valve for preventing pressure rises on power transmission lines due, for instance, to a change in the normal working of the system or to an impulsive rush of electricity caused by a disturbance of the atmospheric potential.

If both the electrodes are of aluminium, then, at an altern-

ating pressure the maximum value of which does not exceed the critical pressure, very little current will pass through the cell, and at a pressure the maximum value of which is, let us suppose, 10 per cent. greater than the critical pressure a very large current will pass. Hence a battery of cells of this type would form a suitable safety valve to be connected between two alternating current mains to prevent the pressure from ever becoming excessive. C. Garrard (*Electrician*, vol. lix, p. 147) states that the critical voltage for a cell having two aluminium electrodes dipping into a bichromate solution is about 110 volts. Hence, for a safety valve between 20,000 volt alternating current mains, about 280 of these cells would be required, if the voltage is sine shaped so that the normal maximum pressure is 28,280 volts.

One effect in connexion with these cells which has to be remembered when they are used on alternating current circuits is that they act as electrostatic condensers. The thickness of the film round the aluminium anodes is microscopic and its resistance is very high. The P.D. across this film is appreciable, and thus the electrostatic charge due to the condenser action is also appreciable. In practice, n of these condensers are connected in series, and thus the resultant capacity of the battery of cells between the mains is only the nth part of the capacity of one cell. Although this capacity is in general very small yet with the very high frequency "pressure rises" sometimes set up by an arc in the circuit, the condenser current may appreciably relieve the pressure.

The electrolytic lightning arrester is generally used in conjunction with a spark gap (Fig. 63). As there is no leakage current in this case, there is no risk of the electrolyte evaporating or of the electrodes being deteriorated by over-

heating. The air-gap also can be adjusted to act within much narrower limits than when an ordinary resistance is used. In Fig. 63, *C* represents a pile of electrolytic elements which in practice would be enclosed in an earthenware pipe.

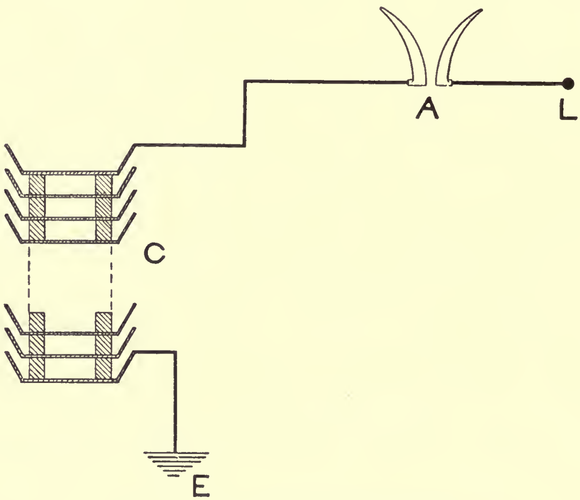

Fig. 63.—Electrolytic Arrester.

Each element consists of a shallow aluminium dish. They are separated from one another by pieces of insulating material. A solution of bichromate of potash is sometimes used for the electrolyte. It is poured in the top dish slowly, filling it, and then trickling down and filling all the other dishes in turn. A drop of transformer oil in each of the trays forms a thin film over the surface of the electrolyte and thus hinders evaporation. When the critical voltage across a battery of this type is exceeded, the surfaces of each dish are covered with tiny sparks and brush discharges, indicating the points where the insulation resistance of the non-conducting film has broken or is breaking down.

In a type of lightning arrester described by R. Jackson

(*The Electric Journal*, vol. iv, p. 469) very similar to that described above (Fig. 63), the voltage required per cell is stated to be 400. Hence if V be the effective voltage of the alternating current between the line and earth, and k the amplitude factor, so that Vk is the maximum value of the voltage the number of cells required would be slightly more than $Vk/400$. If the voltage, for instance, between the line and earth is 10,000, and the pressure wave is sine shaped, the number of cells required would be slightly more than $10{,}000 \times 1{\cdot}414/400$. Hence 40 would be sufficient.

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LIGHTNING CONDUCTORS

CHAPTER XII

Lightning Conductors

Lightning conductors—Atmospheric electricity—Potential gradient of the atmosphere—Kew observations—Thunderstorm potential gradients—The potential difference required for a lightning flash—The A flash—The B flash—Lightning rods—The current in the conductor—Numerical example—Side flash—Lightning Research Committee—Earthing—Tubular earth—The metal of the conductor—The elevation rod—Town houses—Lightning fatalities—References.

Lightning conductors

As lightning conductors for protecting buildings from lightning have now been in use for over a hundred years, there is naturally plenty of information available to illustrate the effects produced when a lightning flash strikes a conductor. It is only, however, comparatively recently, mainly owing to the researches of Sir Oliver Lodge, that a satisfactory theory has been developed to explain the phenomena. We shall first briefly consider the causes of thunderstorms, and then discuss in detail the function of lightning conductors, or as they are frequently called, lightning rods.

Atmospheric electricity

It is universally admitted that a lightning flash is a phenomenon similar to that which ensues when a Leyden jar is discharged by a spark. It is clearly due to electricity in the air. During thunderstorms characteristic black clouds are observed, and the flash takes place between a cloud and the earth or between two

clouds. We may conclude that previous to the flash a high potential difference must exist between the two conductors subsequently short circuited by the flash.

We have next to consider what produces this potential difference between a stratum of the atmosphere and the earth or between two different atmospheric strata. The friction between neighbouring strata moving with different velocities probably developes static charges on the minute drops of water carried along by the air currents. Any alteration of the level of the strata will rapidly alter the potentials of these charges. As the dielectric strength of air is not very great, the electric stress will sometimes cause a disruptive discharge, which may travel considerable distances owing to the violent equalizations of the potentials and consequent increase in the electric stresses between other strata which may ensue as the flash proceeds almost instantaneously from one stratum to another. The energy originally expended, by the sun's heat in vaporizing and raising water to heights in the air, is converted during the flash into heat, light, sound, and electric waves radiating into space.

Potential gradient of the atmosphere

The results obtained by many experiments prove that there is practically always a difference of potential between atmospheric strata which are at different heights, the positive potential normally increasing with the height. From the results obtained in numerous balloon ascents F. Linke (*Meteorologische Zeitschrift*, vol. 22, p. 237) finds that if V be the potential in volts, and h the height in metres above the ground, then, from 1,500 to 6,000 metres (his highest observation)

$$dV/dh = 34 - 0{\cdot}006h.$$
$$= 25 - 0{\cdot}006(h - 1{,}500).$$

The potential gradient of the atmosphere dV/dh, therefore, diminishes the farther we get away from the earth. Up to

1,500 metres he finds that the gradient varies from day to day, but above this height the gradient seems to be practically the same over long periods. If we accept the above formula, the potential at 4,000 metres is nearly 44,000 volts above that at 1,500 metres. If we assume for the average gradient up to 1,500 metres, the mean of that at the ground in Linke's experiments, namely 125 v/m, and that at 1,500 metres (25 v/m), we find that the potential at 1,500 metres equals $75 \times 1{,}500$, that is, 110,000 volts approximately. This gives for the potential difference between a stratum of air 4,000 metres high and the earth about 150,000 volts.

A potential gradient of 125 volts per metre, except during midsummer, is really a low ground value to assume. In winter, if the weather be fine its value is generally about 300 volts per metre, and in foggy weather it is sometimes 1,000 volts per metre. Thus the above estimate is probably often much exceeded.

Kew observations

C. Chree by analysing the readings of the Kelvin water-dropping electrograph at Kew Observatory has found that there are two distinct daily maxima and minima values of the potential gradient. In all months the minima occur near 4 a.m. and 2 p.m. The times at which the maxima occur are more variable. He also finds that the day interval between the forenoon and the evening maximum is longer in summer than in winter.

The month showing the highest mean potential gradient is December, but the amplitude of the diurnal inequality is greatest in February. With the exception of the month of July a high mean potential and a large diurnal range of potential were found associated with a low temperature.

It has to be remembered that these results are strictly applicable to Kew only. It is highly probable that in

mountainous districts the electrical atmospheric phenomena would be different. The tendency, in winter, to a single diurnal period visible at Kew is more pronounced elsewhere.

It is interesting to remember the importance that Kelvin attached to a study of atmospheric electricity. The Kew water-dropping electrograph which Chree used in his observations was probably the first one ever made. Kelvin came to Kew and had it put up under his immediate supervision.

Thunder-storm potential gradients

The values of the potential differences during a thunderstorm have not yet been measured, but the potential gradients near the ground are sometimes at least ten times greater than on ordinary days. In mountainous districts, generally when the air is dry, but sometimes even during rain, brush discharges occasionally take place from pointed objects showing that the potential gradient is very high. The action is sometimes so energetic that a hissing noise is heard.

The black appearance of a thundercloud may be easily imitated by putting a point maintained at a high potential into the steam escaping from a kettle. The effect of electrifying a drop of water is to diminish the value of the hydrostatic surface tension and hence the electrified globules of steam coalesce giving a much darker shade to the cloud of escaping steam.

The effect produced by electrified globules of water coalescing is to raise the potential of the cloud. To prove this, let us consider what happens when n drops, of radius r and at potential v, coalesce into one of radius R and at potential V. Since the volumes and charges remain the same and the capacity of a raindrop is approximately equal to its radius, we have

$$(4/3)\pi R^3 = n(4/3)\pi r^3, \text{ and } VR = nvr.$$

Hence, $$R^3 = nr^3, \text{ and } V^3R^3 = n^3v^3r^3.$$

We have therefore

$$V^3 = n^2 v^3, \text{ or } V = n^{2/3} v.$$

Hence the potential of the large drop is $n^{2/3}$ times that of the smaller drops, and the stress V/R at its surface equals $n^{1/3}(v/r)$.

To obtain some idea of the potentials called into play let us consider the disruptive voltages between large spherical electrodes. In the following table (*see* Chapter VIII) x denotes the minimum distance in metres between the equal spherical electrodes whose radius is stated, and V is the disruptive pressure in kilovolts.

Radius.	1 cm.	10 cm.	100 cms.	1,000 cms.
x	V	V	V	V
0·05	61	163	187	191
0·1	—	280	370	381
0·5	—	604	1,625	1,860
1·0	—	—	2,795	3,690
5·0	—	—	6,030	16,200
10·0	—	—	—	28,000
50·0	—	—	—	60,000

For instance, when the potential difference between two spherical conductors each 1 metre in radius and 1 metre apart attains the value of 2,795 kilovolts, then, under normal atmospheric conditions the air between them will be broken down. The blank spaces in the above table refer to cases where brush discharges ensue before the disruptive discharge. The values of the disruptive voltages, in these cases, depend on the rate at which the ionization of the air surrounding the

electrodes is proceeding. It is probable therefore that their values can only be roughly predetermined.

In the table of the sparking voltage between needle points published by the American Institution of Electrical Engineers and quoted on p. 179 it will be seen that 24·4 cms. is given as the sparking distance for 100 kilovolts alternating pressure or for 141·4 kilovolts direct. From the above table we see that 163 kilovolts will spark across 5 cms. when the radius of the electrodes is 10 cms. and consequently the distance between their centres is 25 cms. It will thus be seen that if we suppose the spherical electrodes to shrivel up into minute spheres having the same centres as the original electrodes an appreciably smaller voltage will suffice to break down the dielectric.

The potential difference required for a lightning flash

Lightning flashes have been observed more than two miles long and the potential differences required previous to the discharge must be considerable. If we assumed that a mean electric stress of about 100 kilovolts per inch is necessary, then about 13,000,000 kilovolts would be required to produce a flash two miles long. This number fixes a superior limit to the value of the voltage necessary to produce this flash. If the flash were to occur in dry clear weather between a cloud two miles high and the earth, and the air between the two was not appreciably ionized, the pressure required might possibly be about 10,000,000 kilovolts. As however, in England at least, lightning flashes practically always occur during rain or hail storms, it is probable that a much smaller voltage suffices. We have already seen that on a clear day the voltage at a height of 4,000 metres is usually at least 150 kilovolts. During a thunderstorm it is probably at times much higher. As a first rough approximation we may conclude that the voltage between an ordinary thunder-

cloud and the earth, immediately before a discharge, lies in value between 100 and 1,000,000 kilovolts.

The A flash

Sir Oliver Lodge divides lightning flashes roughly into two main classes which he calls the A and the B flash respectively. These flashes produce very different effects and it is necessary to distinguish carefully between them. The A flash is illustrated in Fig. 64. In this case the difference of potential between the cloud and the earth gradually increases until the air between them breaks down owing to the great electric stress to which it is subjected and a disruptive discharge ensues which diminishes appreciably the potential difference between the cloud and the earth. The distinguishing characteristic of the A flash is the previous gradual building up of the voltage between the cloud and the earth. Immediately before the flash occurs the potential gradient at all points on earthed conductors is very steep. Round these points the air is being ionized at a rapid rate and the stream of ionized air from them forms a path of small resistance for the disruptive discharge.

FIG. 64.—The *A* Flash.

Lodge has devised the following simple and instructive experiment (Fig. 65) to illustrate the phenomena connected with the A flash. C and E are metal plates insulated from one another. They are connected with the terminals of a Wimshurst frictional machine W, and a Leyden jar L is placed as a shunt between the plates so as to increase the intensity

of the discharge when it occurs. Model lightning conductors consisting of metallic knobs of various sizes and shapes on conducting supports are placed on the lower plate. We may consider that C represents the cloud and E the earth.

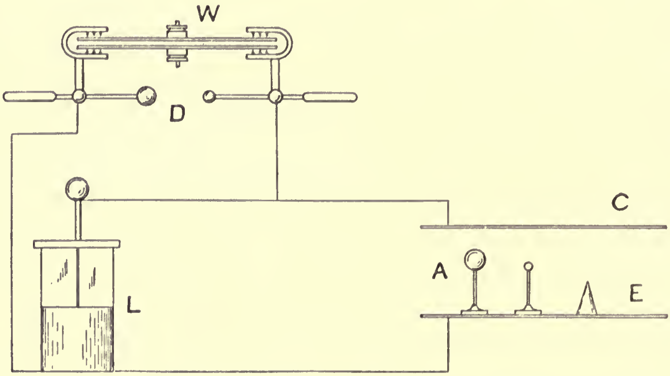

FIG. 65.—Model illustrating the laws governing the *A* Flash.

On turning the handle of the Wimshurst machine a difference of potential is gradually established between C and E. As soon as the potential gradient at a point of any of the conductors exceeds the dielectric strength of the air between the plates there will either be a disruptive spark between the conductor and C, or there will be a brush discharge from the conductor. If there be only two model conductors on E, and if the centre of the small knob of one be closer to C than the centre of the large knob of the other, it will in general protect it, that is, the disruptive discharge will take place between the smaller knob and C, even when the minimum distance between the small knob and C is very appreciably greater than that between the large knob and C.

It is found that the resistance of the supporting pieces connecting the knobs with the lower plate has very little

effect on their liability to be struck. For instance, when the stands are of metal and a damp cloth is placed between the stand of the small knob and the lower plate, the small knob is still struck even although the resistance of its connexion with the lower plate is hundreds or thousands of times greater than that of the larger knob.

With the arrangement shown in Fig. 65, model pointed conductors are so effective in dissipating the charge that it is almost impossible to obtain a spark at all when they are used. If a lighted gas burner be placed on the lower tray

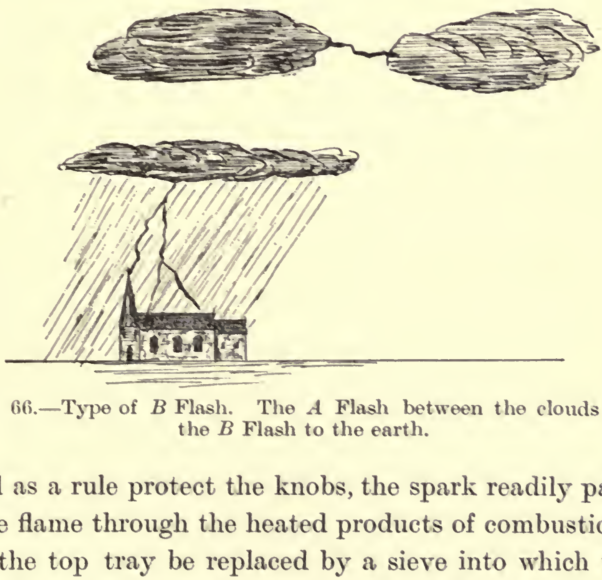

FIG. 66.—Type of *B* Flash. The *A* Flash between the clouds causes the *B* Flash to the earth.

it will as a rule protect the knobs, the spark readily passing to the flame through the heated products of combustion.

If the top tray be replaced by a sieve into which water is poured, it is impossible to obtain a spark at all.

The B flash The B flash which is caused by an impulsive rush of electricity occurs when the difference of potential between the cloud and the earth is established almost instantaneously. There are several varieties of this flash. In Fig. 66, for example, a discharge between two clouds alters by electrostatic induction the potential differ-

ence between another cloud and the earth, and this voltage being greater than the air can withstand, we have a B flash between the cloud and the earth.

An experimental illustration of this flash is shown in Fig. 67. As in Fig. 65, *C* and *E* are two sheets of metal represent-

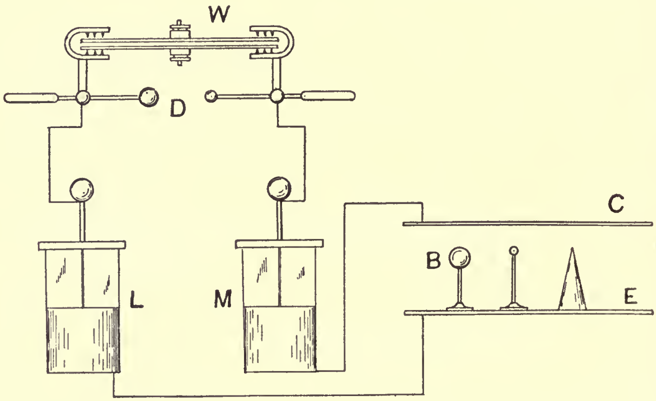

Fig. 67.—Model illustrating the action of the *B* Flash.

ing a cloud and the earth, *L* and *M* are two Leyden jars which we suppose to be placed on a badly insulating wooden table. Their inner coatings are connected with the discharge knobs *D* of a Wimshurst machine *W*, and the outer coatings are in metallic connexion with *C* and *E*.

On turning the handle of the Wimshurst machine, the inner coatings are brought to a high difference of potential, and there will be large electrostatic charges induced on the outside coatings of the jars at these high potentials. When a spark occurs at *D* the potentials of the inner coatings will be equalized, probably by an oscillating discharge, and the potential difference between the outside coatings of *L* and *M*, owing to the large charges on them of equal and opposite sign, will attain a high value. Hence also the potential difference between *C* and *E* (Fig. 67) which are in metallic

connexion without the outside coatings will be high, and if the height of C above the model lightning conductors be not too great there will be a spark discharge. Before the spark occurs at D, the potential difference between the plates is very small, as each is practically at earth potential, for the inductive effects produced by the equal and opposite charges on the inner coatings of the Leyden jars practically neutralizes the effects produced by the outer coatings. But when the spark occurs at D the potential difference between C and E is altered practically instantaneously, the spark between them is therefore of the B type.

FIG. 68.—A second type of B Flash.

In this case the action of the model lightning conductors is quite different to their action in the preceding case (Fig. 65). In Fig. 67, for example, where the cone and the tops of the knobs are all of the same height the B spark takes place between C and one or other of these conductors. If one be placed slightly nearer to the upper plate than the others it will protect them.

Other varieties of the B flash are illustrated in figs. 68 and 69. An experimental illustration of these cases is shown in Fig. 70. When a spark takes place between the discharge knobs D, a B flash will pass between the highest lightning conductor and C provided that the distance between them be not too great. The shape of the end of the lightning conductor is quite immaterial in this case, the protective

action being quite different to that which occurs with A flashes. There is no time for the ionization of the air which takes place at points to prepare a path of small resistance for the discharge. The rush of electricity apparently always takes place across the shortest path. Lodge compares the paths in the steady stress and in the impulsive rush cases to the paths taken down a hill side by a gentle stream of water and by an avalanche respectively.

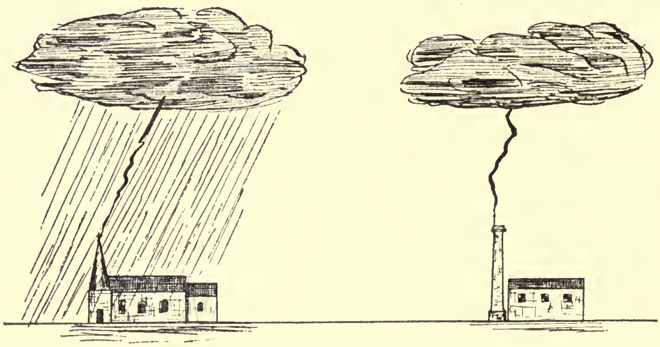

FIG. 69.—A third type of *B* Flash. The *A* Flash from a cloud to the chimney stack causes the *B* Flash from a neighbouring cloud.

As in the case of the A flash it is found that the absolute values of the resistances of the lightning conductors themselves have little effect on their protective qualities.

If we replace the top sheet of metal in Figs. 67 and 70 by a sieve into which water is poured, sparks still ensue. The flashes, like those which occur during thunderstorms, are sometimes very long and very irregular. They seem to make use of the rain drops as stepping stones.

As thunderstorms in this country are nearly always accompanied by rain, it is highly probable that most of the flashes which occur belong to the impulsive rush case. In the majority of cases also it is probable that the discharge is oscillatory, for we know both by theory and experiment

that the spark discharge of a condenser is oscillatory provided that the resistance of the path of the discharge be not above a certain value.

Lightning rods The main function of a lightning rod is to dissipate the energy stored in the lightning flash harmlessly, and so prevent it from doing damage to neighbouring objects. Hence the conductor must not be too small in diameter or it will be deflagrated by the discharge.

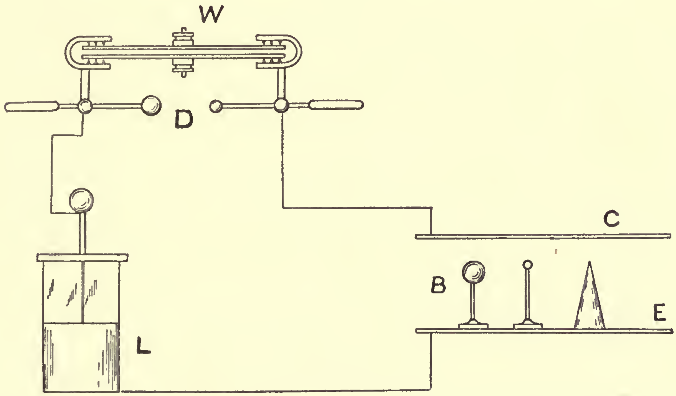

FIG. 70.—Model illustrating second type of *B* Flash.

A subsidiary function is to equalize the potential between the thunder cloud and earth by the "silent discharge" taking place from all points on the conductor. This action is probably less energetic than the discharging action of certain kinds of trees—for instance, fir trees. Statistics prove that the cutting down of extensive fir forests in certain parts of Europe has led to a considerable increase in the number of destructive lightning flashes experienced in those districts. It is probable, therefore, that in towns where numerous lightning conductors with multiple points on them are employed, they will have the effect of diminishing the average number of the lightning flashes that occur.

The current in the conductor

When the discharge is oscillatory it must have an exceedingly high frequency, and when it is non-oscillatory the discharge is over in a very small fraction of a second. In either case we know from theory that the current in the lightning conductor flows so that the magnetization in the metal of the conductor is a minimum (*cf*. Chapter II, p. 43). It is therefore practically confined to a thin layer of the metal on the outer surface of the conductor. In calculating the resistance of the path, therefore, we must not, as in electric light wiring, proceed on the assumption that the current density is uniform over the cross section.

Lord Rayleigh has shown that when the frequency is very high the resistance R and the self-inductance L of a cylindrical rod for a symmetrical flow of current, obeying the sine law, are given by

$$R=(l/a)\sqrt{\rho\mu f}, \quad \text{and} \quad \mathrm{L}=l\{A+(1/2\pi a)\sqrt{\rho\mu/f}\}.$$

In these equations l denotes the length of the conductor, a its radius, ρ the resistivity in absolute units, μ the permeability of the metal, f the frequency, and A a constant depending on the dimensions, etc., of the return circuit. All the quantities in the equations are in C.G.S. units.

Numerical example

Let us suppose that the lightning conductor is a cylindrical copper rod 100 metres long and 1 centimetre in diameter. In this case $l=10{,}000$, $a=0{\cdot}5$, $\rho=1{,}600$ approximately, and $\mu=1$. We shall suppose also that the frequency is 1,000,000, so that $f=10^6$. Substituting these values in the formula for R we find that $R=(10^4/0{\cdot}5)\sqrt{1{,}600\times10^6}=8\times10^8$ absolute units $=0{\cdot}8$ of an ohm. The resistance to a flow of direct current would be 0·02 of an ohm. Hence the resistance of the lightning conductor to an impulsive rush of electricity is forty times as great as that which it would offer to direct currents or

alternating currents of the frequencies used for electric lighting.

If the conductor had been made of iron, all the dimensions remaining the same, and if we take the resistivity of iron as nine times that of copper and assume that its average permeability under the given conditions is 100, then, R will equal 24 ohms, and the resistance with direct current will be 0·18 of an ohm which is less than the hundredth part of the apparent resistance to the alternating impulsive rushes.

It follows from Rayleigh's formula that the inductance of the conductor is $lA + R/2\pi f$. Hence the reactance is $2\pi f lA + R$. In most cases R will be very small compared with $2\pi f lA$, and hence the inductance and reactance of lightning conductors is practically independent of the material of which they are made.

Rayleigh's formulae show that the greater the radius of a cylindrical rod the smaller will be its resistance and inductance. As the radius of the rod increases, however, the greater will be the ratio of the apparent resistance R_a with alternating currents to the resistance R_d with direct currents, for $R_a \,/\, R_d = \{(l \,/\, a)\sqrt{\rho\mu f}\}/(\rho l/\pi a^2) = \pi a \sqrt{\mu f/\rho}$. Hence the ratio R_a/R_d varies directly as the radius of the rod, but the absolute value of R_a diminishes as a increases.

It is to be remembered that the longer the conductor, or the greater its resistance, the lower will be the frequency of the oscillations set up by the lightning flash. We have also to remember that in calculating the values of R no account has been taken of the energy lost by radiation into space, which at these high frequencies is probably appreciable.

Side flash

If a piece of wire is placed sufficiently close to a lightning rod, part of the charge will leave the rod and travel along the piece of wire as the reactance of the divided path is less than the path in the conductor

alone. This explains the phenomenon of side flash which is often observed when an object is struck by lightning. The potential differences existing between various parts of a lightning conductor when it is struck are obviously very high and hence the electrostatic field round it is very intense. The electric stresses ionize the air round the conductor and so a spark readily ensues to any neighbouring conductor. In setting up lightning conductors this tendency to side flash has to be remembered, as sparks due to this cause can ignite escaping gas and thus set fire to buildings. It has often been noticed that when a lightning rod is struck a peculiar noise is heard not unlike the pouring of water on a fire, and electric sparks are emitted from bodies in the neighbourhood. These phenomena are probably caused by brush discharges due to the breaking down of the air by the electrostatic stresses set up during the discharge.

Lightning Research Committee

The Lightning Research Committee, appointed by the Royal Institution of British Architects and the Surveyors' Institution in 1901, have in their report made the following practical suggestions.

1. Two main lightning rods, one on each side should be provided, extending from the top of each tower, spire or high chimney-stack by the most direct course to earth. The diagrams shown in Fig. 71 illustrate this suggestion.

In *Y*, which is the usual method, the conductor follows the outline of the building. In this case there is a tendency for the discharge to leave the conductors at the bends, as it always tends to make a path or paths for itself in addition to that provided by the lightning rod, so that the reactance of all of them in parallel may be a minimum. It thus sometimes breaks away the brickwork, and in some cases the mechanical forces called into play break the conductor

itself. The Research Committee recommend the method illustrated in *X* (Fig. 71), where the conductor is kept away from the building by suitable holdfasts, which may be made of iron.

It seems to the author that the method *X* recommended is excellent for getting round sharp corners, or in cases where there is danger from side flash owing to the presence of neighbouring conductors. In general, however, when there

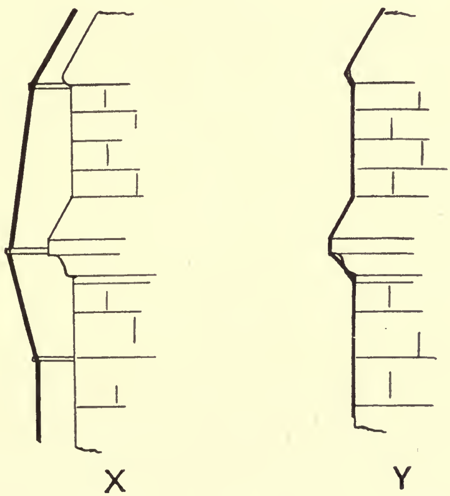

FIG. 71.—X is the method of fixing lightning conductors recommended by the Lightning Research Committee.

is a straight run for the conductor there is no need to keep it away from the surface of the wall.

2. Horizontal conductors should connect all the vertical rods (*a*) along the ridge, (*b*) at or near the ground line.

This recommendation of the Committee was probably suggested to obviate the risks of side flash from one conductor to the other and as a partial protection also for the space between the two.

3. The upper horizontal conductor should be fitted with aigrettes or points at intervals of 20 or 30 feet.

4. Short vertical rods also should be erected along minor pinnacles, and connected with the upper horizontal conductor.

5. All roof metals such as finials, ridging, rain water and ventilating pipes, metal cowls, lead flushing, gutters, etc., should be connected with the horizontal conductors.

6. All large masses of metal in the building should be connected with the earth, either directly or by means of the lower horizontal conductor.

7. Where roofs are partially or wholly metal-lined they should be connected with the earth by means of vertical rods at several points.

8. Gas pipes should be kept as far away as possible from the positions occupied by lightning conductors, and as an additional protection the service mains of the gas meter should be metallically connected with house services leading from the meter.

Many useful suggestions will also be found in the Report issued by the Lightning Rod Conference held in 1882. Some of the rules given in this report, however, have to be amended as they proceed on the erroneous assumption that a lightning flash will follow the path of minimum resistance in exactly the same way that a steady direct current would.

Earthing

The end of the lightning conductor is usually connected with a copper plate embedded in moist earth in the neighbourhood of the building. If none of the earth in the immediate neighbourhood of the conductor is moist, it is advisable to dig a pit about 6 feet deep in which the sheet of copper about a square yard in area and one-eighth of an inch thick should be placed and then surrounded with charcoal or pulverized carbon. The ends

of the carbons used in arc lamps do excellently for this purpose. Coke is sometimes employed, but its use is objectionable owing to the chemical and electrolytic effects produced in the copper. The pit should not be quite filled up with earth, so that there may be a sufficient depression on the surface over the pit to catch the rain during a thunderstorm and thus keep the earth in the neighbourhood of the plate moist.

The resistance of the "earth" is measured by finding, by a Wheatstone's bridge or otherwise, the resistance between the conductor and any neighbouring water pipe. On a dry day if this resistance be not greater than 100 ohms the "earth" may be considered satisfactory. Accurate measurements of this resistance are neither possible nor necessary. In the neighbourhood of towns supplied with electric light or tramways a permanent deflection is often obtained on the galvanometer owing to a leakage current from some of the supply networks. Instead of using a special "earth" it is sometimes convenient to connect the end of the lightning conductor with the water mains.

Tubular earth

Mr. Killingworth Hedges' "tubular earth" can often be advantageously used. It consists of a hollow perforated steel spike filled with granulated carbon and driven into moist earth. The lightning conductor is taken to the bottom of the tube. The earth in the neighbourhood of this device can easily be kept moist by connecting it with the nearest rain-water pipe. In this case the earth resistance is negligibly small.

The metal of the conductor

The lightning conductors used in this country are generally made of copper. Either copper tape or copper wire rope is employed. In the former case the section is usually $\frac{3}{4}$ inch broad by $\frac{1}{8}$ inch thick, this size being found ample in practice. In

the latter case the rope is usually ½ inch in diameter. If smaller sized conductors are used there is a risk of them being deflagrated by a severe lightning flash.

In those climates where there is little risk of the conductor being corroded either by the moisture or by chemical fumes, galvanized soft stranded iron rope is the most suitable lightning conductor. The higher specific heat of an iron conductor compensates for its smaller density, and so its higher melting point enables it to get rid of a larger amount of the electrical energy of the flash than a copper conductor of the same dimensions.

The elevation rod

The elevation rod or top of a lightning conductor ought to be the highest point of the building, but there is no necessity to have it more than about a foot taller than the summit of a pinnacle or the brickwork of a chimney. Four or five well gilded or platinized "points" should be attached to the elevation rod.

Town houses

There is not much danger of town houses being struck by lightning, as the numerous gutters, ventilating and rain-water pipes afford them considerable protection. Occasionally metallic bonds are used to connect the various sections of rain-water pipes, and thus ensure their metallic continuity and so guard against damage by lightning. As most fire insurance policies issued in England cover damage done by lightning, these precautions are seldom taken for ordinary town houses. Important buildings are usually elaborately protected by lightning conductors. Even with very elaborate systems, however, possible dangers arise from side flash from the conductors to neighbouring gas pipes or stove pipes. The *Hôtel de Ville* at Brussels, which is protected by a very complete network of wires, had a narrow escape from

being burned down during a thunderstorm, as a spark from one of the lightning conductors to a neighbouring piece of metal set fire to gas which had escaped from a leak in a gas pipe.

Lightning fatalities

Heavy damp soils such as loam are particularly liable to be struck by lightning flashes. The most frequent fatalities in this country from lightning happen to people standing under trees which are struck, the lightning "side flashing" from the tree to the person whose body, or clothes if wet, forms a good conductor. Trees whose roots are near the water are particularly liable to be struck. Again a person in the centre of a field or crossing the brow of a hill might possibly be struck, as he would be the highest object in the neighbourhood. Horses, cattle, and sheep, especially when steam is rising from them owing to their being overheated, are sometimes struck. Deer in public parks are frequently killed owing to their habit of congregating under trees during thunderstorms. In America wire ropes are often used to hang clothes on to dry after being washed. Several fatalities occur every year to people taking the clothes off these ropes at the beginning of a thunderstorm.

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